Impact of stratification and climatic perturbations to stratification on barotropic tides

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Collaborators

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- Peter Miller (*U-Michigan*)

Outline

1. Observations of secular changes in tides
2. Exploration of potential causes
   - Changes in water column thickness (Müller et al. 2011)
   - Changes in stratification (this talk)
3. Results from global numerical models
4. Analytical model
   - Governing equations
   - Solution with imposed mean flow (scattering)
   - Solution with imposed astronomical forcing
   - Results
5. Conclusion

Observations of secular changes in tides

- Cartwright (1971, 1974)
- Woodworth et al. (1991)
- Flick et al. (2003)
- Ray (2006, 2009)
- Colosi and Munk (2006)
- Jay (2009)
- Woodworth (2010)
- Müller et al. (2011)

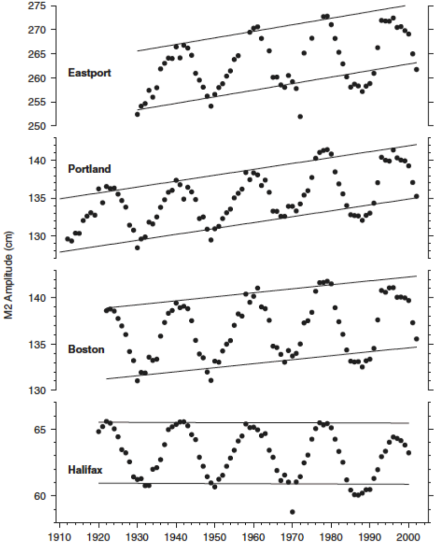

Fig. 4. Yearly estimates of the amplitude of the M_2 tide at four tide-gauge stations. Straight lines delineate upper and lower extrema of a pre-1980 least-squares fit to each dataset, with functional form of a bias plus trend plus 18.6-y sinusoid.

Observations of secular changes: Müller et al. (2011)

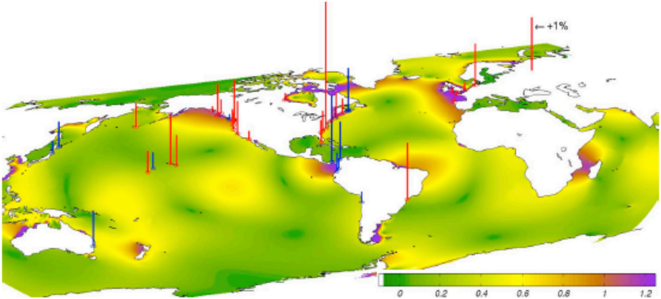

Figure 1. Fractional trends in M_2 amplitude. The reference bar on the Eurasian continent shows a trend of 1% decade^{-1}. Red (blue) bars denote positive (negative) trends. Color contours provide the tidal amplitude in m (TPXO.7.2) [*Egbert et al.*, 1994].

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- Work in progress is utilizing HYCOM which is non-Boussinesq, and a more sophisticated analysis of stratification changes

Exploration of potential causes

Why should the tide respond to changes in the water column thickness and stratification?

Exploration of potential causes: (Gill 1982)

For a two-layer system with no damping the phase speeds c of the two modes are given by

$$c^4 - g(H_1 + H_2)c^2 + gg'H_1H_2 = 0$$

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We'll attempt to answer this question in this study

Global baroclinic tide model

Work with the HIM tide model builds upon Arbic et al. (2004).

The model has:

- M_2 forcing
- Tuned parameterized topographic wave drag
- $1/8°$ resolution from $86°$S to $82°$N
- Horizontally uniform two-layer stratification
  - Control interface depth at 700 m
  - Control value of $g' = g(\rho_2 - \rho_1)/\rho_2$ is 1.64×10^{-2} m s^{-2}
- Self-attraction and loading (Hendershott 1972) computed with full spherical harmonic treatment in iterative manner

Observed climatic changes in stratification

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- Some locations show
  - Isopycnal displacements of 100 dbars per century
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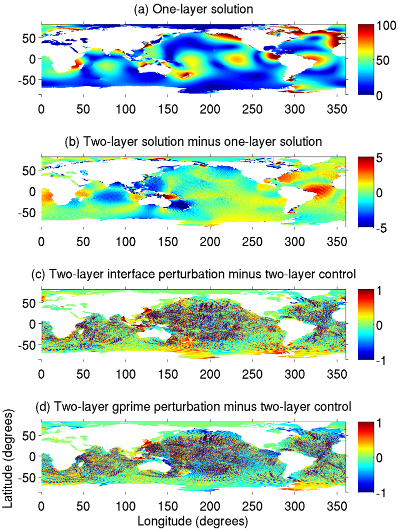
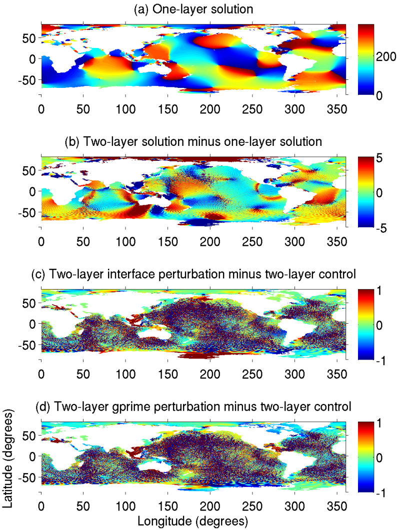
Currently working on a more sophisticated methodology which arrives at similar perturbation values

Results from global HIM

Amplitude (cm)



Phase (degrees)



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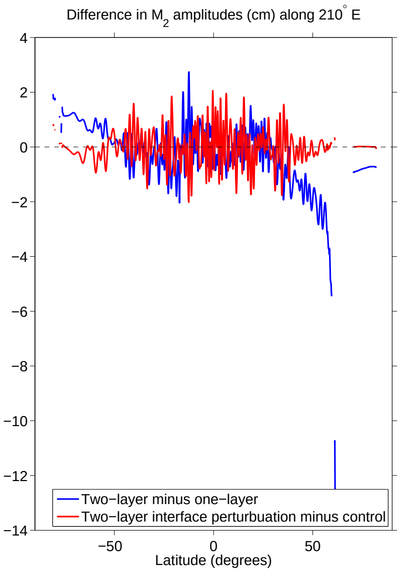
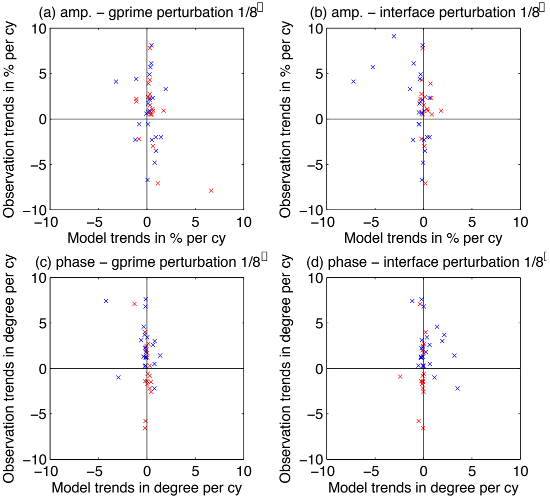


Comparison: global HIM vs. data



Analytical model

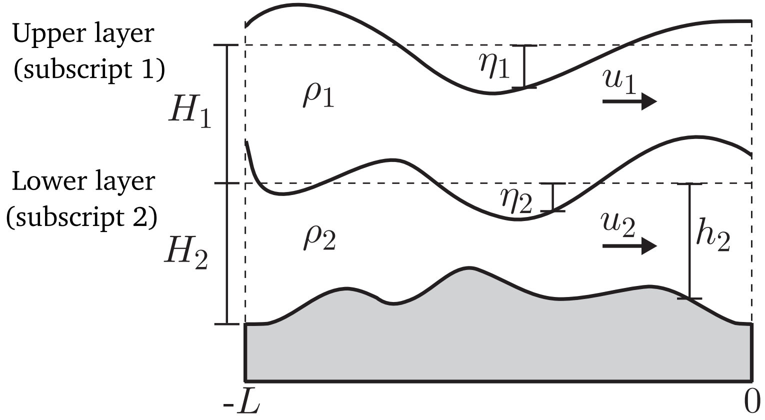


Figure : Sketch of one-dimensional analytical two-layer model.

Analytical model

Variables:

- $h_2(x)$ is the bottom layer resting depth at a given location
- $\eta_1(x,t)$ and $\eta_2(x,t)$ are the layer perturbation displacements
- $u_1(x,t)$ and $u_2(x,t)$ are the layer velocities

Parameters:

- H_1 and H_2 are the largest values of the resting layer depths
- ρ_1 and ρ_2 are the layer densities
- r_1 and r_2 are the layer damping rates
- g is gravity and g' is the reduced gravity
- L is the scale of the basin

Governing equations

Conservation of Mass:

$$\frac{\partial}{\partial t}(\eta_1 - \eta_2) + H_1 \frac{\partial u_1}{\partial x} = 0,$$

$$\frac{\partial \eta_2}{\partial t} + \frac{\partial}{\partial x}\left[h_2 u_2\right] = 0,$$

Conservation of Momentum:

$$\frac{\partial u_1}{\partial t} = -g\frac{\partial \eta_1}{\partial x} - r_1 u_1 + g\frac{\partial}{\partial x}\eta_0 e^{i(kx+\omega t)},$$

$$\frac{\partial u_2}{\partial t} = (g' - g)\frac{\partial \eta_1}{\partial x} - g'\frac{\partial \eta_2}{\partial x} - r_2 u_2 + g\frac{\partial}{\partial x}\eta_0 e^{i(kx+\omega t)},$$

with boundary conditions

$$u_1(-L,t) = u_2(-L,t) = u_1(0,t) = u_2(0,t) = 0.$$

Results

For the separable solution

$$\eta_1(x,t) = N_1(x)e^{i\omega t}, \quad \eta_2(x,t) = N_2(x)e^{i\omega t}, \ldots,$$

the system can be reduced to a set of non-constant second order ODEs for the velocities. These can be solved in a natural way using Green's function theory.

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For arbitrary topography we obtain:

- Sensitivity of barotropic and baroclinic scales to stratification
- Separation of the solution into barotropic and baroclinic modes

Results

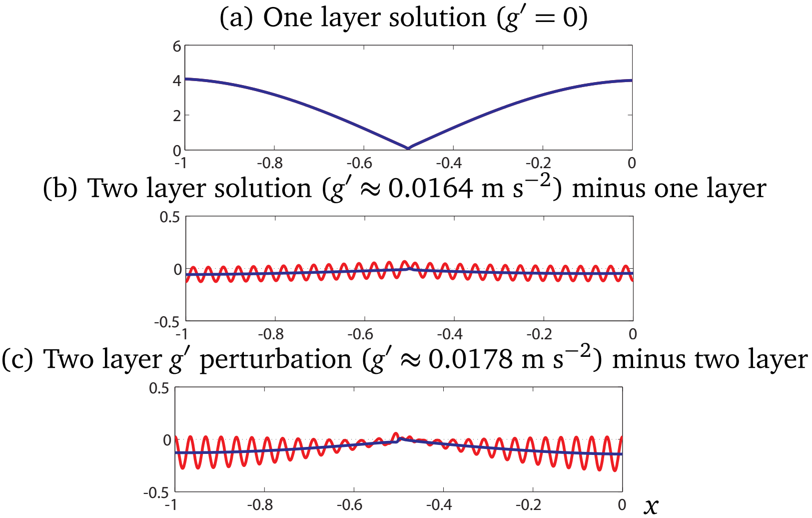


Figure : Surface elevation (red line) and barotropic surface elevation (blue line) amplitudes in a finite basin.

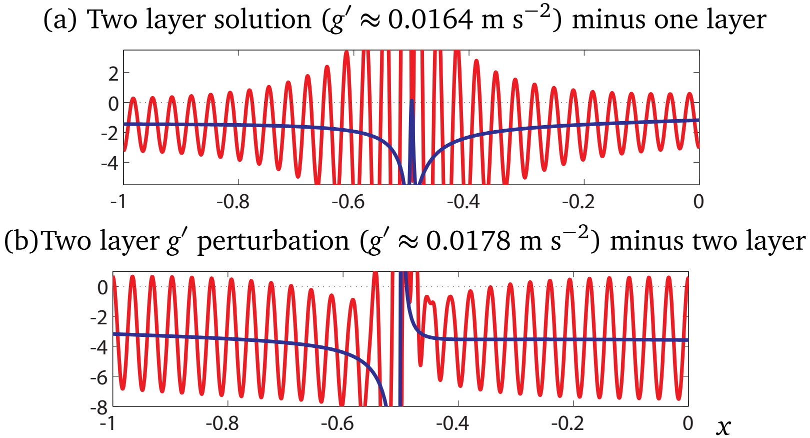
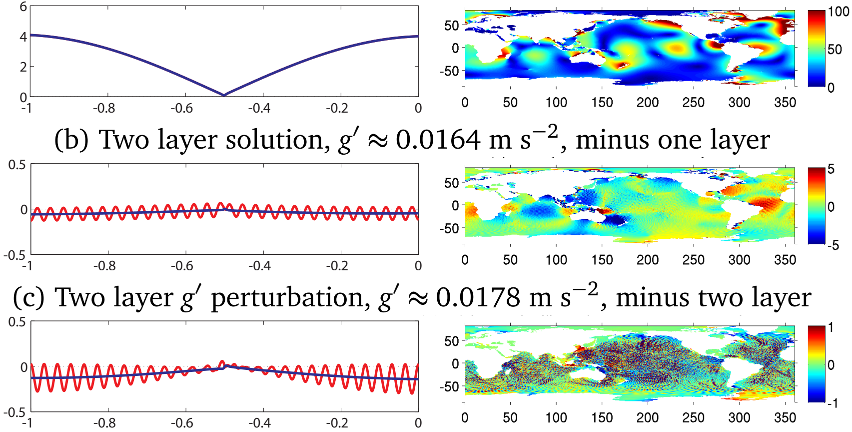


Figure : Surface elevation (red line) and barotropic surface elevation (blue line) amplitudes in a finite basin as a percentage of the control solution.

Results

(a) One layer solution, $g' = 0$

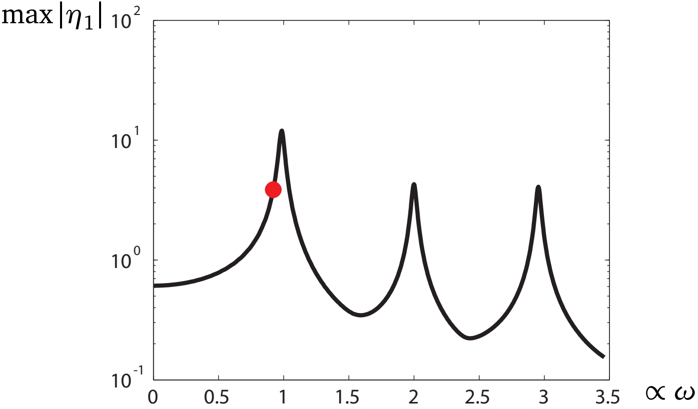


(b) Two layer solution, $g' \approx 0.0164$ m s^{-2}, minus one layer

(c) Two layer g' perturbation, $g' \approx 0.0178$ m s^{-2}, minus two layer

Likely explanation for this sensitivity.

Resonance graph for analytical model



Our chosen realistic parameters place us (red dot) very close to a resonance peak

Conclusion

- Addition of stratification changes surface elevation amplitudes by 1%-5%, and phases by 1°-5° at large and small scales
- Additional changes in the stratification **either in interface depth or value of g'** yield comparable perturbations in the barotropic and baroclinic modes

These results are consistent in both the numerical model and analytical model.

In other words, changes in the surface tidal elevations from perturbations on oceanic stratification occur not only in the expected small scale (baroclinic) component, but also in the large scale (barotropic) component.

Future work

- Investigate the importance of the Boussinesq approximation
  - Current analytical model is non-Boussinesq while HIM is Boussinesq
- Include the effects of shelf/open-ocean coupling in analytical model
  - Analytical model does not include shelf, where secular changes have been observed
- More layers in analytical and numerical models
  - Better representation of coastal stratification

Thank You