Combining HYCOM, AXBTs and Polynomial Chaos Methods to Estimate Wind Drag Parameters during Typhoon Fanapi

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Outline

The Problem
Drag Parameterization
Bayesian formulation of inverse problem

The Tools
Polynomial Chaos

The results
PC Analysis
The inference posteriors
Variational Solution

Conclusions
\[ \vec{\tau} = \rho_a C_D \vec{V} \vec{V} \]
\[ C_D = C_{D0} + C_{D1} (T_s - T_a) \]
\[ C_{D0} = a_0 + a_1 \tilde{V} + a_2 \tilde{V}^2 \]
\[ C_{D1} = b_0 + b_1 \tilde{V} + b_2 \tilde{V}^2 \]
\[ \tilde{V} = \max \left[ V_{\text{min}}, \min (V_{\text{max}}, V) \right] \]

\( C_D \) is drag coefficient
\( V \) is wind speed at 10 m.
\( C_D \) saturates for \( V > V_{\text{max}} \)

- Blue circles: aircraft observations (French et al., 2007),
- red: wind tunnel (Donelan et al., 2004),
- green: drop sondes (Powell et al., 2003),
- magenta: HYCOM fit to COARE 2.5,
- Problem: \( V_{\text{max}} \) and \( C_{D_{\text{max}}} \) are not well-known and does \( C_D \) decrease for \( V > V_{\text{max}} \) as drop sondes suggest?
Inverse Modeling Problem

- Perturb $C_D$ by introducing 3 control variables ($\alpha$, $V_{\text{max}}$, $m$)

  \[ C_D' = \alpha C_D \text{ for } V < V_{\text{max}} \]  
  \[ C_D' = \alpha [C_D + m(V - V_{\text{max}})] \text{ for } V > V_{\text{max}} \]  

- multiplicative factor $0.4 \leq \alpha \leq 1.1$
- vary $V_{\text{max}}$ between 20 and 35 m/s
- $m$ is a linear slope modeling decrease for $V > V_{\text{max}}$ with $-3.8 \times 10^{-5} \leq m \leq 0$
- Use ITOP data to learn about likely distribution of $\alpha$, $V_{\text{max}}$ and $m$. 
Bayes Theorem: $p(\theta \mid T) \propto p(T \mid \theta) \ p(\theta)$

- Likelihood: $\epsilon = T - M$ is normally distributed

\[
p(T \mid \theta) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi \sigma^2}} \exp \left( -\frac{(T_i - M_i)^2}{2\sigma^2} \right) \tag{3}\]
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(3)

- \( \sigma^2 \) unknown, treated as hyper-parameter. Assume a Jeffreys prior

\[
p(\sigma^2) = \begin{cases} 
  \frac{1}{\sigma^2} & \text{for } \sigma^2 > 0, \\
  0 & \text{otherwise}.
\end{cases}
\]

(4)
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  \]  
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- Uninformed priors for \( \alpha \), \( V_{\text{max}} \) and \( m \):
  \[
p(\{\alpha, V_{\text{max}}, m}\}) = \begin{cases} \frac{1}{b_i-a_i} & \text{for } a_i \leq \{\alpha, V_{\text{max}}, m}\} \leq b_i, \\ 0 & \text{otherwise}, \end{cases}
  \]  
  (5)

where \([a_i, b_i]\) denote the parameter ranges.
Final Form of Bayes theorem

\[ p(\{\alpha, V_{\text{max}}, m\}, \sigma^2 | T) \propto \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(T_i - M_i)^2}{2\sigma^2}\right) \left[ p(\sigma^2) p(\alpha) p(V_{\text{max}}) p(m) \right] \]
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- Build full posterior with Markov Chain Monte Carlo (MCMC)
  MCMC requires \( O(10^5) \) estimates of \( M_i \): prohibitive
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- Solve for center and spread of posterior
  minimization problem requiring access to cost function gradient and Hessian: Needs an adjoint model
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- Rely on Polynomial Chaos expansions to replace HYCOM by a polynomial series that could be either summed for MCMC or differentiated for the gradients.
What is Polynomial Chaos

- Series Representation of Model Output

\[ M(x, t, \theta) = \sum_{k=0}^{P} M_k(x, t) \psi_k(\theta) \]  

- \( M(x, t, \theta) \): a model output (aka observable)
- \( M_k(x, t) \): series coefficients
- \( \psi_k(\theta) \): orthogonal basis functions w.r.t. \( p(\theta) \)
- mean: \( E[M] = \langle M, \psi_0 \rangle = \sum_{k=0}^{P} M_k(x, t) \langle \psi_k, \psi_0 \rangle = M_0(x, t) \)
- Variance: \( E \left[ (M - E[M])^2 \right] = \sum_{k=1}^{P} M_k^2(x, t) \)

- Basic Questions
  - How to choose \( \psi_k \)? Legendre polynomials
  - How to determine the coefficients \( M_k \)? Projection
  - Where to truncate the series, \( P \)? Monitor Variance
How do we determine PC coefficients

• Series: \( M(x, t, \theta) = \sum_{k=0}^{P} M_k(x, t) \psi_k(\theta) \)

• Projection:

\[ M_k(x, t) = \langle M, \psi_k \rangle = \int M(x, t, \theta) \psi_k(\theta) \rho(\theta) \, d\theta \]

• Approximate integral with numerical Quadrature

\[ M_k(x, t) \approx \sum_{q=1}^{Q} M(x, t, \theta_q) \psi_k(\theta_q) \omega_q \]

• \( \theta_q/\omega_q \) quadrature points/weights

• Quadrature requires an ensemble run at \( \theta_q \)

• Here we Used Adaptive quadrature requiring 6-iteration levels for a total of 67 realizations
**Figure:** Fanapi’s JTWC track (black curve) and paths of C-130 flights. The yellow circles on the track represent the typhoon center at 00:00 UTC. The circles on the flight paths mark the 119 AXBT drops. The $42 \times 42$ km$^2$ analysis box is also shown.
Figure: Comparison of HYCOM vertical temperature profiles with AXBT observations on Sep 14 (left), 15 (center) and 17 (right). Temperature averages over the first 50 m are shown in the legend.
PC Representation Errors

Evolution of the area-averaged SST realizations (blue) and of the corresponding PC estimates (red). The normalized rms error (right panel) remains below 0.1% for the duration of the simulation.
Figure: Normalized error between realizations and the corresponding PC surrogates at different depths; Top row: 00:00 UTC Sep 15; bottom row: 00:00 UTC Sep 18.
Depth Profile of Temperature Statistics

50m-deep mixed layer
2°C cooling after Fanapi arrives
Uncertainties confined to top 50 m.
SST Response Surface

Figure: SST response surface as function of $\alpha$ and $V_{\text{max}}$, with fixed $m = 0$. Plots are generated on different days, as indicated. SST's dependence on $V_{\text{max}}$ decreases after 09/17.
Markov Chain Monte Carlo

Figure: Top row: chain samples for $V_{max}$, $m$ and $\alpha$. Bottom row: chain samples for $\sigma^2$ generated for different days, as indicated.
Figure: Posterior distributions for the drag parameters (top) and the variance between simulations and observations (bottom). The numbers show the Kullback-Liebler divergence quantifying the distance between 2 prior and posterior pdfs, i.e. the information gain.
Remarks on posteriors

• $V_{\text{max}}$ exhibits a well-defined peak at 34 m/s.
• Posterior of $m$ resembles prior. Data added little to our knowledge of $m$.
• $\alpha$ shows a definite peak at 1.03 with a Gaussian like-distribution.
• $\sqrt{\sigma^2}$ is a measure of the temperature error expected. This error grows with time from about 0.75$^{\circ}$ to 1$^{\circ}$C.
Figure: Left: joint posterior distribution of $\alpha$ (left) and $V_{\text{max}}$; right: joint posterior of $\alpha$ and $\sigma^2$, generated for Sep 17-Sep 18. Single peak located at $V_{\text{max}} = 34 \text{ m/s}$ and $\alpha = 1.03$. The posterior shows a tight estimate for $\alpha$ with little spread around it.
Figure: Optimal wind drag coefficient $C_D$ using MAP estimate of the three drag parameters. The symbols refer to AXBT data used in the Bayesian inference.
Variational Form

- maximize the posterior density, or equivalently, minimize the negative of its logarithm

\[ J(\alpha, V_{\text{max}}, m, \sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2) = \sum_{d=1}^{5} \left[ J_d + \left( \frac{n_d}{2} + 1 \right) \ln(\sigma_d^2) \right], \quad (7) \]

where \( J_d \) is the misfit cost for day \( d \), the \( \ln(\sigma_d^2) \) terms come from the normalization factors of the Gaussian likelihood functions and from the Jeffreys priors.

- The expression for \( J_d \) is:

\[ J_d(\alpha, V_{\text{max}}, m, \sigma_d^2) = \frac{1}{2\sigma_d^2} \sum_{i \in I_d} [M_i - T_i]^2, \quad (8) \]

where \( I_d \) is the set of \( n_d \) indices of the observations from day \( d \).
Adjoint-Free Gradients

Minimization requires cost function gradients

\[
\left[ \frac{\partial J}{\partial \alpha}, \frac{\partial J}{\partial V_{\text{max}}}, \frac{\partial J}{\partial m} \right] = \sum_{d=1}^{5} \frac{1}{\sigma_d^2} \left( \sum_{i \in \mathcal{I}_d} (M_i - T_i) \left[ \frac{\partial M_i}{\partial \alpha}, \frac{\partial M_i}{\partial V_{\text{max}}}, \frac{\partial M_i}{\partial m} \right] \right)
\]

Compute them from PC expansion

\[
\left[ \frac{\partial M}{\partial \alpha}, \frac{\partial M}{\partial V_{\text{max}}}, \frac{\partial M}{\partial m} \right] = \sum_{k=0}^{P} \hat{M}_k(x, t) \left[ \frac{\partial \psi_k}{\partial \alpha}, \frac{\partial \psi_k}{\partial V_{\text{max}}}, \frac{\partial \psi_k}{\partial m} \right].
\]

- \( \frac{\partial \psi_k}{\partial \alpha} \) easy to compute
- No adjoint model needed
- For Hessian just differentiate above again.
Figure: Posterior probability distributions for (top) drag parameters and (bottom) variances $\sigma_d^2$ at selected days using variational method and MCMC. The vertical lines correspond to the MAP values from MCMC and optimal parameters found using the variational method.
Conclusions & Future Work

- Identified drag parameters from ITOP observations during typhoon Fanapi.
- PC instrumental to make calculations tractable either through MCMC or through adjoint-free minimization
- $V_{\text{max}} \approx 34 \text{ m/s}$
- Data uninformative regarding decrease in $C_D$
- $C_D^{\text{max}}$ peaking around $2.3 \times 10^{-3}$
- Surface temperature measurements more valuable than ones at depths $> 75 \text{ m}$.
- Inference of $V_{\text{max}}$ and $m$ hampered by lack of observation at wind speeds $> 35 \text{ m/s}$.
- Future: Hurricane Model & other air-sea exchange coefficients
Publications


Bibliography

