Global distribution of diurnal and semi-diurnal Parametric Subharmonic Instability in a Global Ocean Circulation Model

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Outline

- Motivation
- Results
  - PSI (Parametric Subharmonic Instability) of the Diurnal & Semi-Diurnal Tides
  - Estimates of fraction of power in subharmonics
  - Rates of energy transfer

Motivation

What's the fate of mode 1 waves that radiate away?

Mode 1 Waves → Black Box → Dissipation

- break on continental shelfs; other topography
- PSI (Parametric Subharmonic Instability) is a culprit:
  $\omega_0 \to \omega_1 + \omega_2, \quad \omega_1, \omega_2 \approx \omega_0/2$

$(\omega_1, \omega_2) \to$ small vertical scales $\Longrightarrow$ mixing

MacKinnon & Winters (2005), Hibiya et al.(1998, 2002), MacKinnon et al. (2011, 2012), Furuichi et al.(2005), Alford(2008), Hazewinkel & Winters (2011), Simmons(2008), Sun (2010), Sun & Pinkel (2013)

Previous Global Simulations on PSI

Simmons (2008) simulations observed PSI in a $1/8^\circ$global model with 16 layers:

(1) Only M_2 tides (2) No wind forcing

(3) Horizontally-uniform stratification

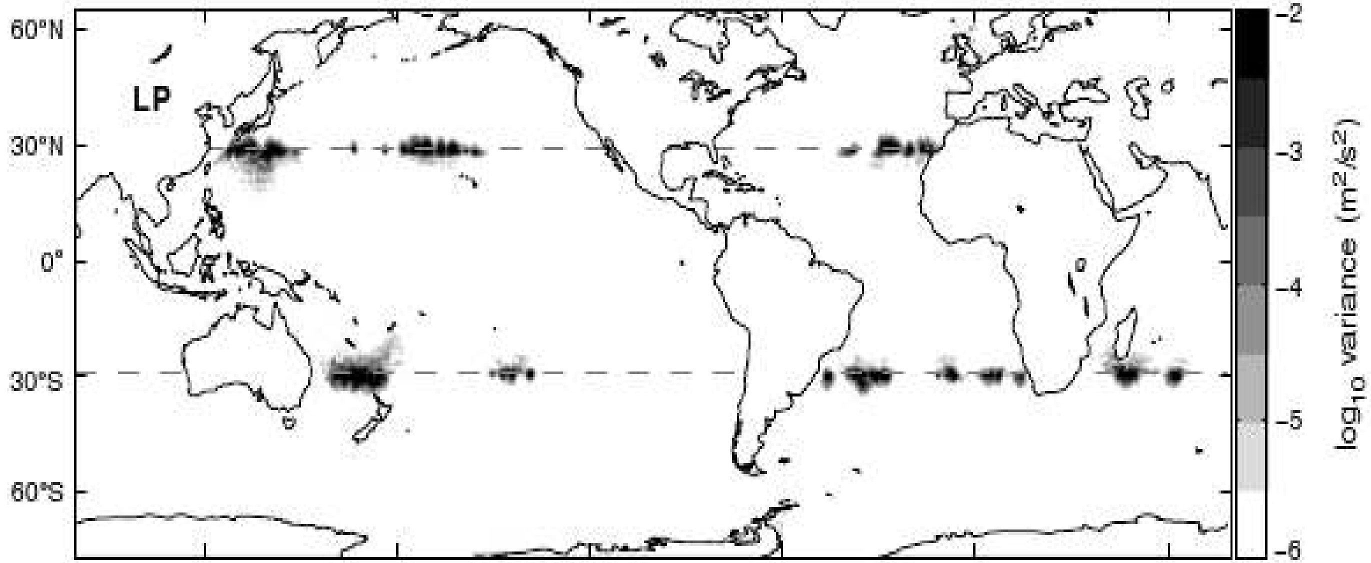


Fig. 3. Geographic distribution of upper ocean baroclinic velocity variance [var(u') + var(v')] averaged over days 160–170. The upper panel shows the time-averaged total variance ("Full"). The bottom panel shows frequencies lower than M_2 ("LP"). The SHTL is indicated by the dashed lines along ±28.8° in the bottom panel.

Simmons (2008)

Low-passed signal

PSI around ...

Critical Latitudes

Present Study

HYCOM simulations with

(Arbic *et al.* 2010, 2012; Richman *et al.* 2012; Shriver *et al.* 2012; Timko *et al.* 2012)

- The 8 major diurnal/semi-diurnal constituents
- Wind forcing (3-hourly from NOGAPS)
- Horizontally-varying stratification

HYCOM grid + Output (experiment 18.5)

- $1/12^\circ$ model, 32 Vertical layers
- 30 days of $3D$ global output

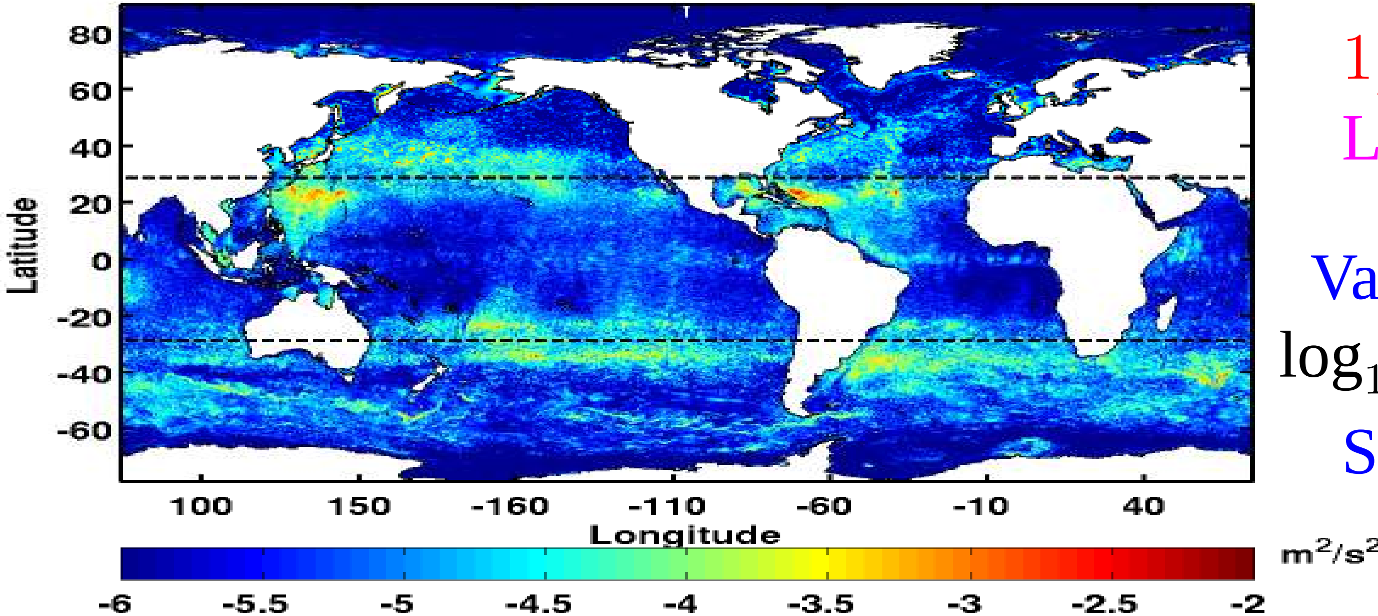
Present Study

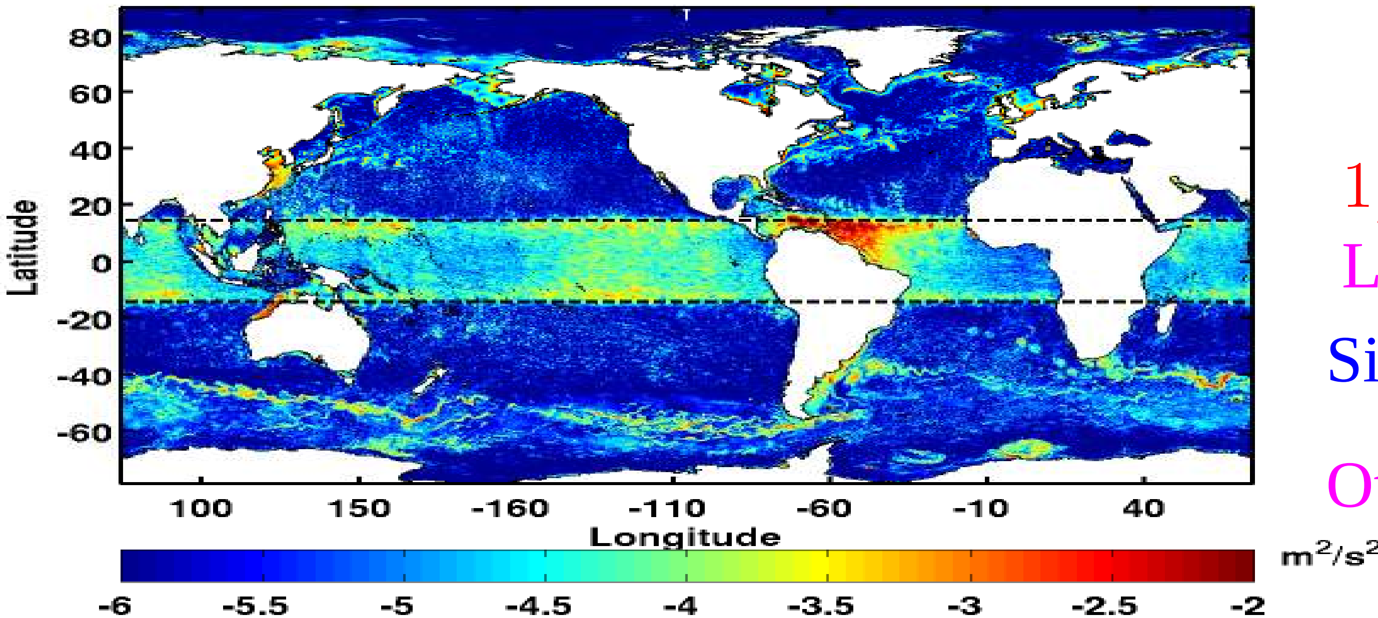
QUESTIONS:

- How would wind-generated NIWs change results...
- Could the eddies alter PSI behaviors?
- Can we detect PSI of diurnals? (Alford 2008)
- How much energy is in subharmonics

START SIMPLY ...

Initial Results





$1/2$ Semi-diurnals

Layer 14 ($\approx 500m$)

Variance:

$\log_{10}[var(u') + var(v')]$

Spread around...

Critical Latitudes

Activity around strong...

WBC + ACC

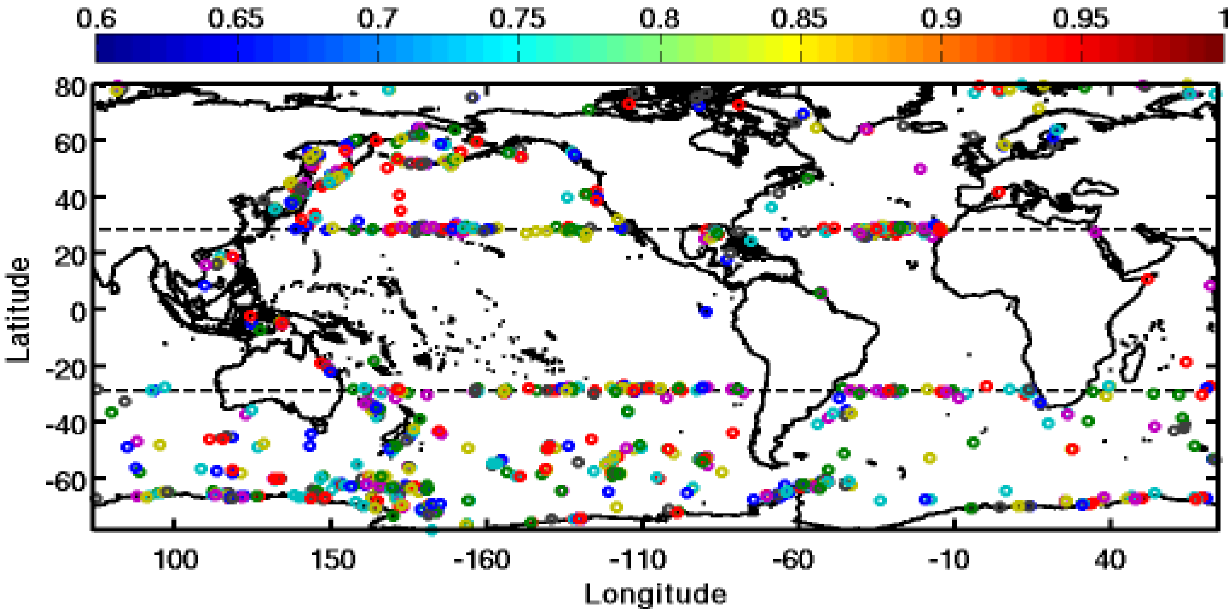
$1/2$ Diurnals

Layer 14 ($\approx 500m$)

Similar story...

Other layers?

Bicoherence Results



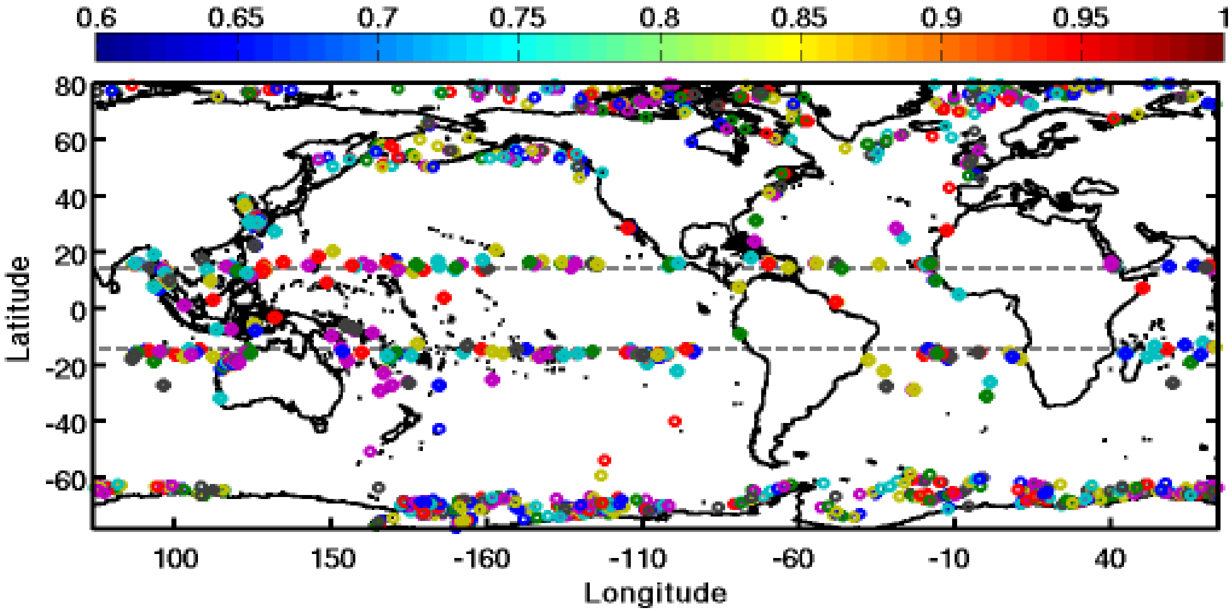
[Kim & Powers 1979]

Semidiurnals

Baroclinic U and V

Layer 14 ($\approx$ 500 m)

Noisy in high latitudes



Diurnals

Noisy in high latitudes

Bicoherence: Noisy/Weak Signals?

- Comment from Ocean Sciences (Tim Duda):
  It is possible in some locations to have signals with high bicoherence but the energy in those signals might be small compared to the tide
- Interpretation? Back to equations (Kim & Powers, 1979):

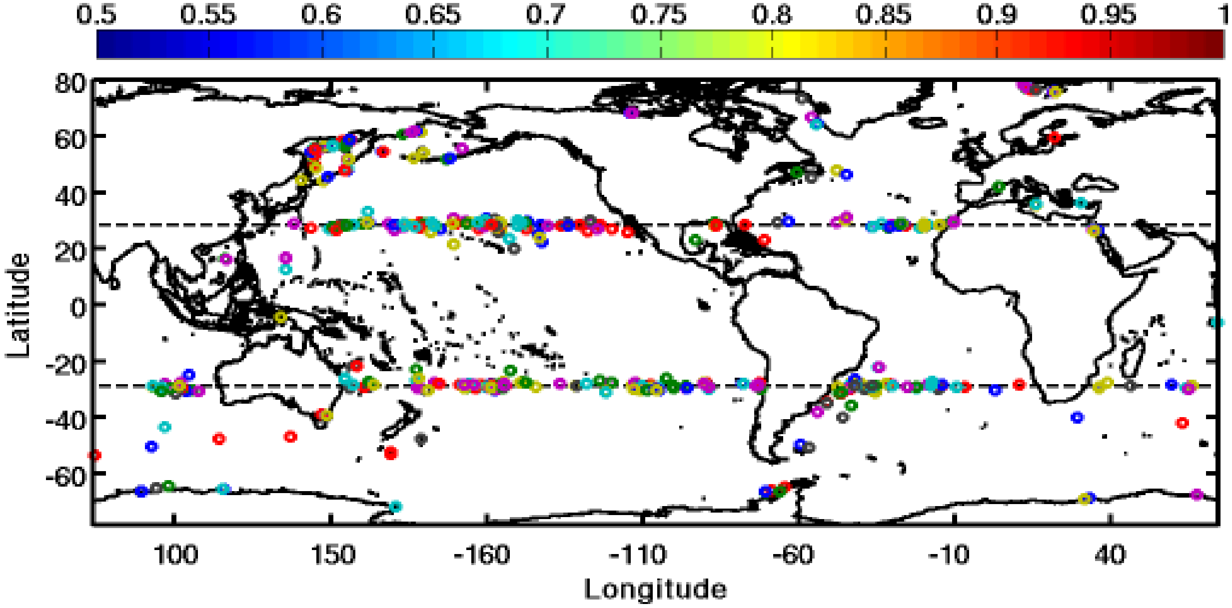
$$b = \frac{|B(\omega_1,\omega_2)|}{\{E[|X_{\omega_1}X_{\omega_2}|^2]\}^{1/2}\{E[|X_{\omega_3}|^2]\}^{1/2}}, \quad 0 \le b \le 1$$

- Define ‘energy ratio’, R:

$$R = \frac{\{E[|X_{\omega_1}X_{\omega_2}|^2]\}^{1/2}}{\{E[|X_{\omega_1}X_{\omega_2}|^2]\}^{1/2} + \{E[|X_{\omega_3}|^2]\}^{1/2}}$$

Eliminate signals with $R < 1\,\%$ and get...

Bicoherence Results

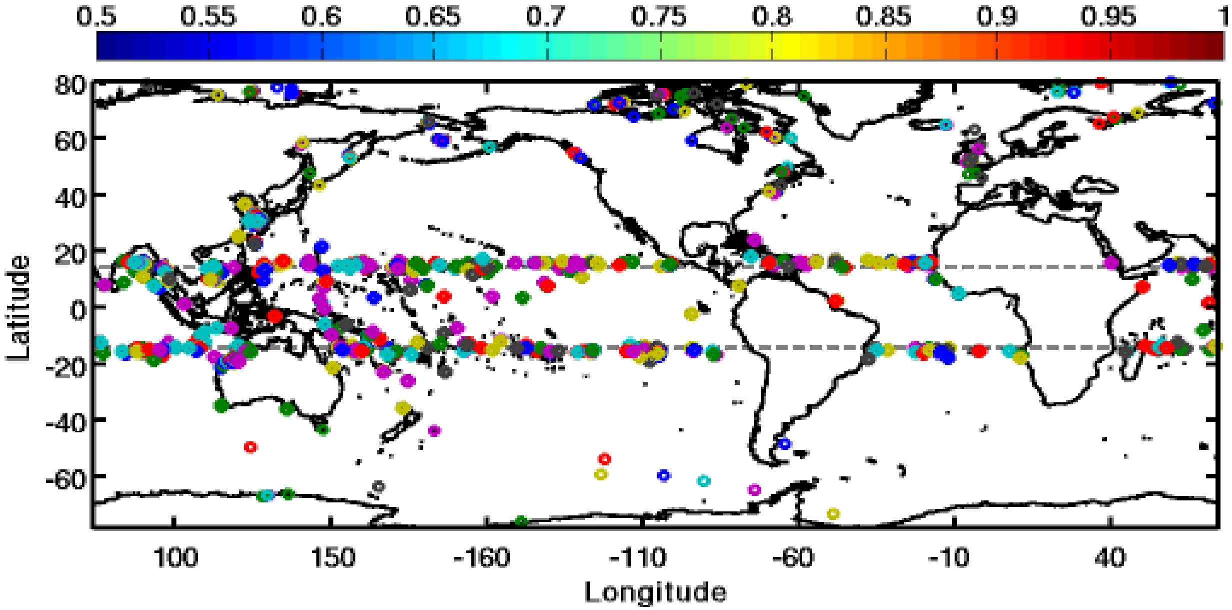


[Kim & Powers 1979]

Semidiurnals

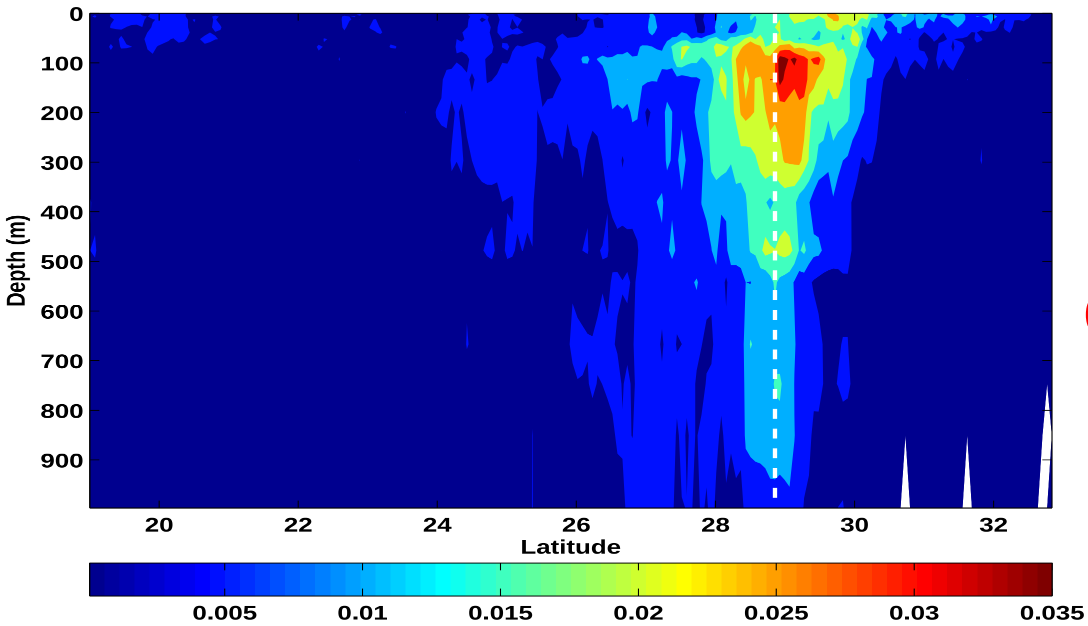
Much cleaner results:

95 % of 'noise' vanish...
in high latitudes



Diurnals

Depth vs Zonally-averaged bicoherence



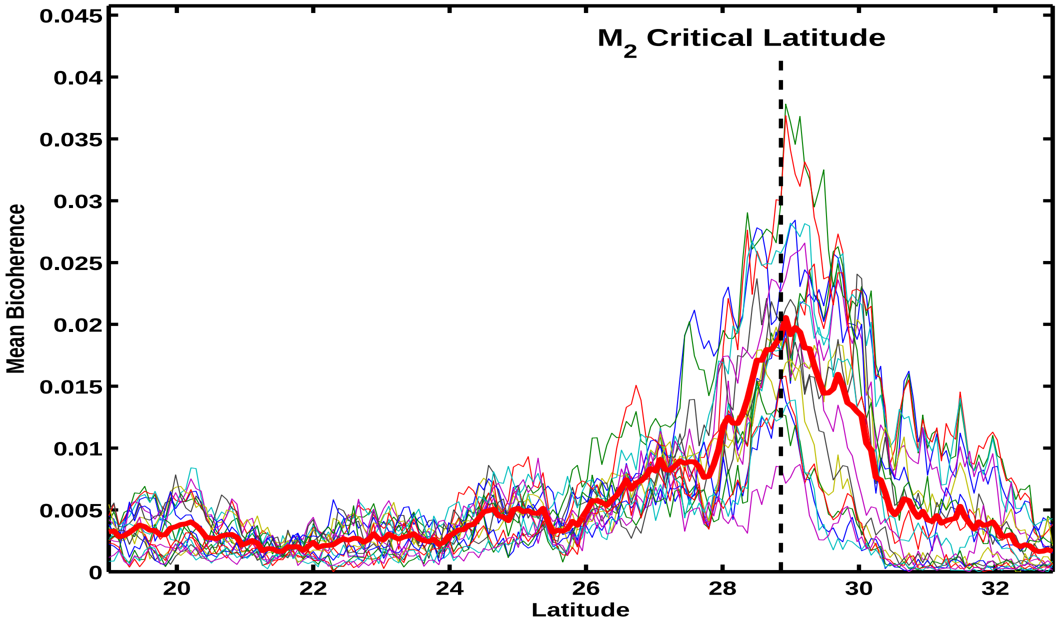
Semi-Diurnal PSI

Variation in depth of zonally-averaged bicoherence

Only showing $0 - 1000\,\mathrm{m}$

Intense in upper ocean

[Hazewinkel & Winters (2011)]

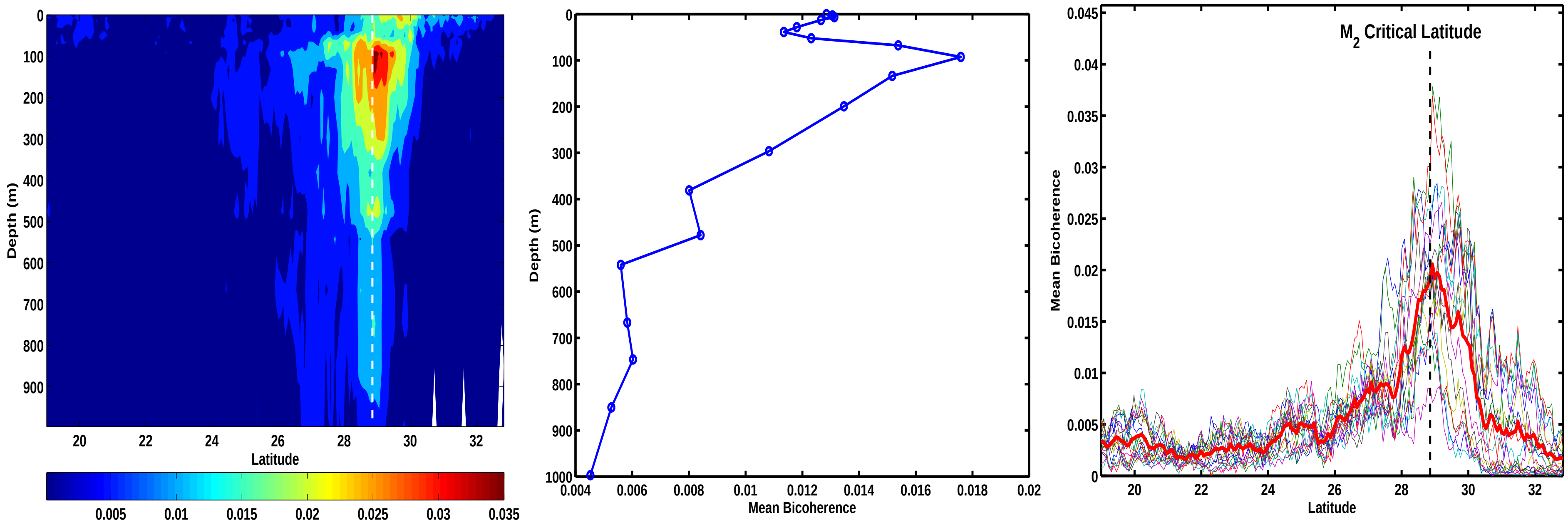


Highest bicoherences centered... on M_2 Critical Latitude

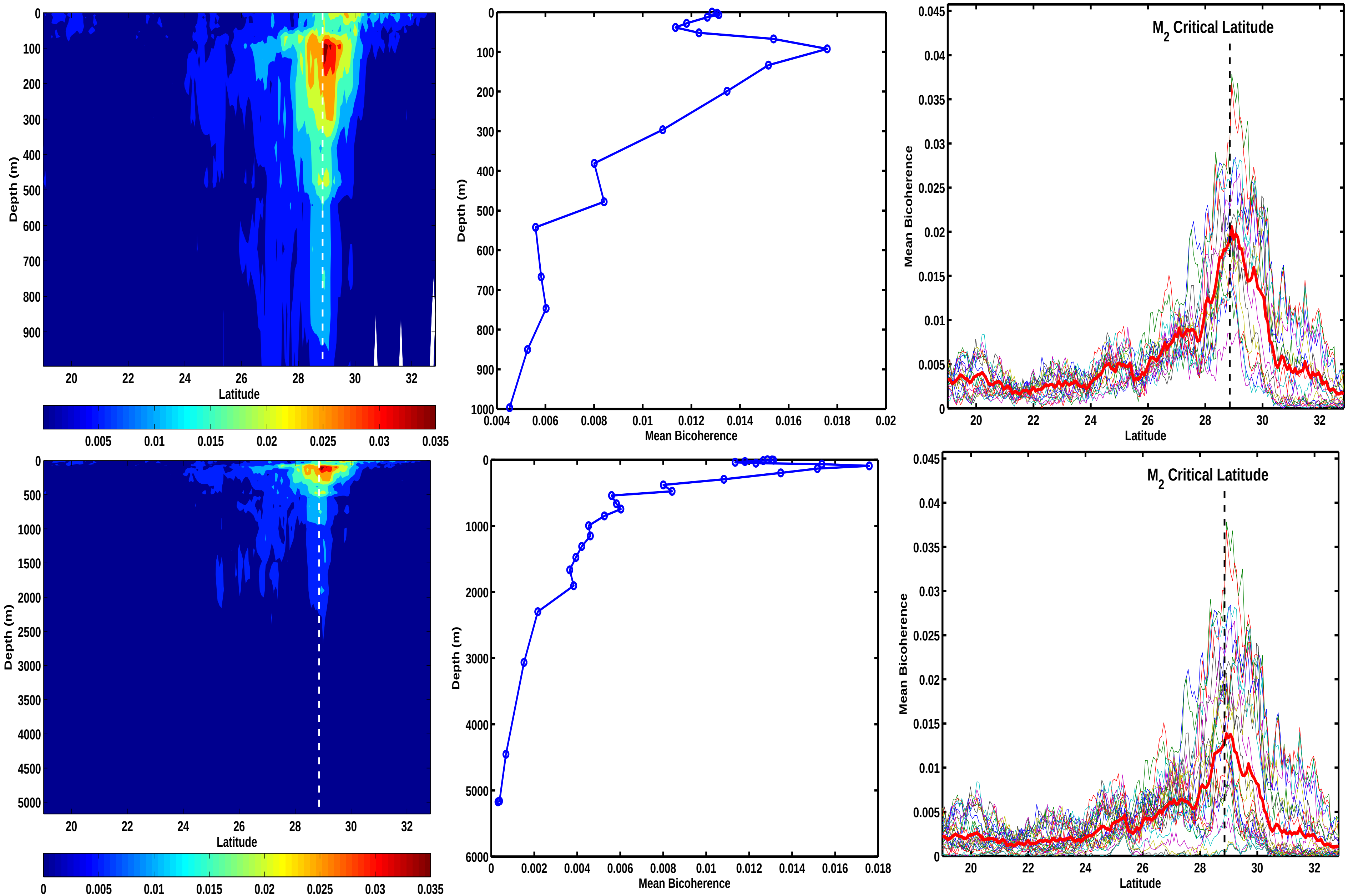
Intensity falls around $\sim 3°$.. from CL

[Furuichi et al. (2005)]

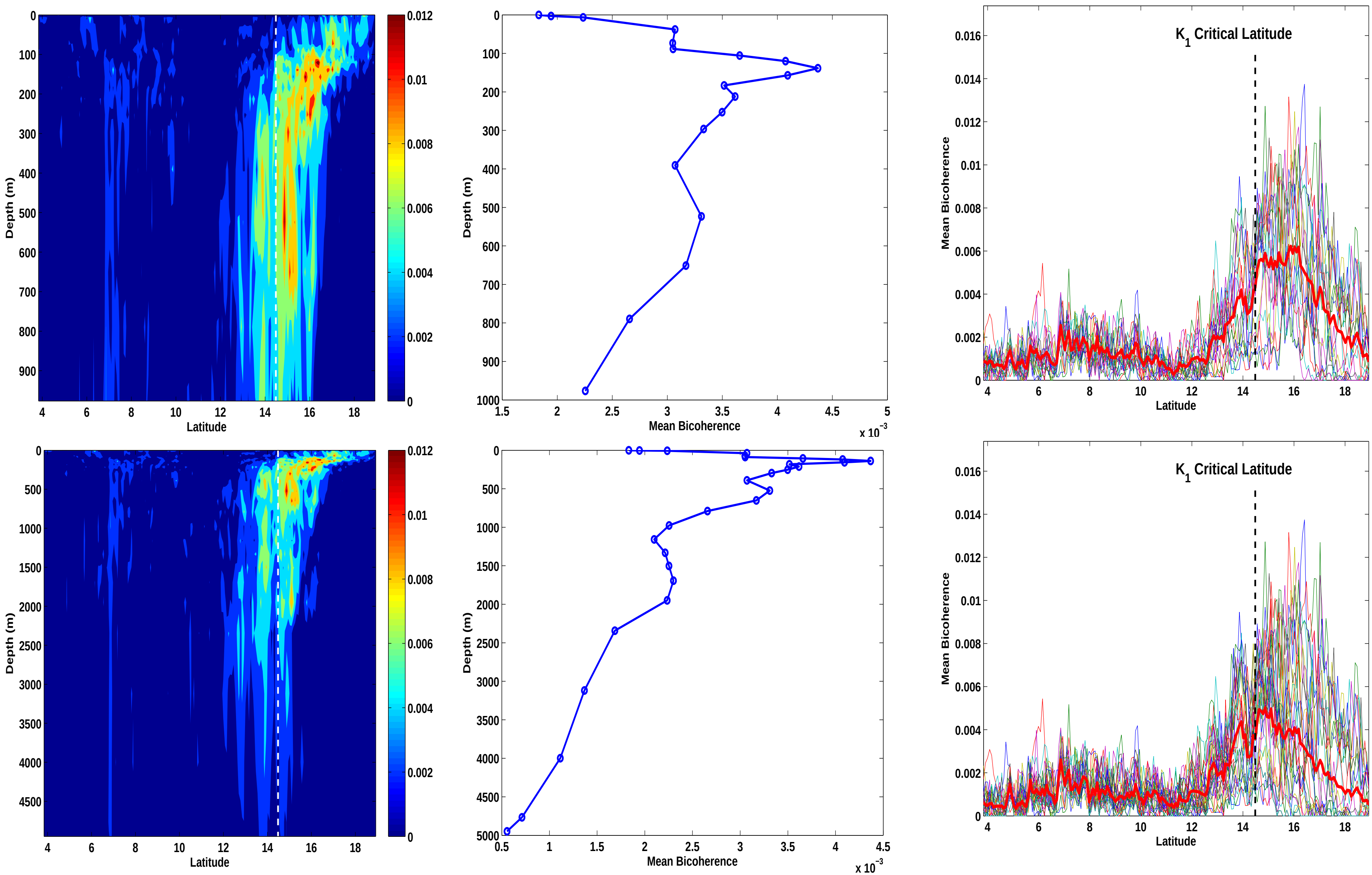
Depth vs Zonally-averaged bicoherence



Depth vs Zonally-averaged bicoherence



Bicoherence: Diurnals



Power in Subharmonics

BIG QUESTION: How much energy is in subharmonics?

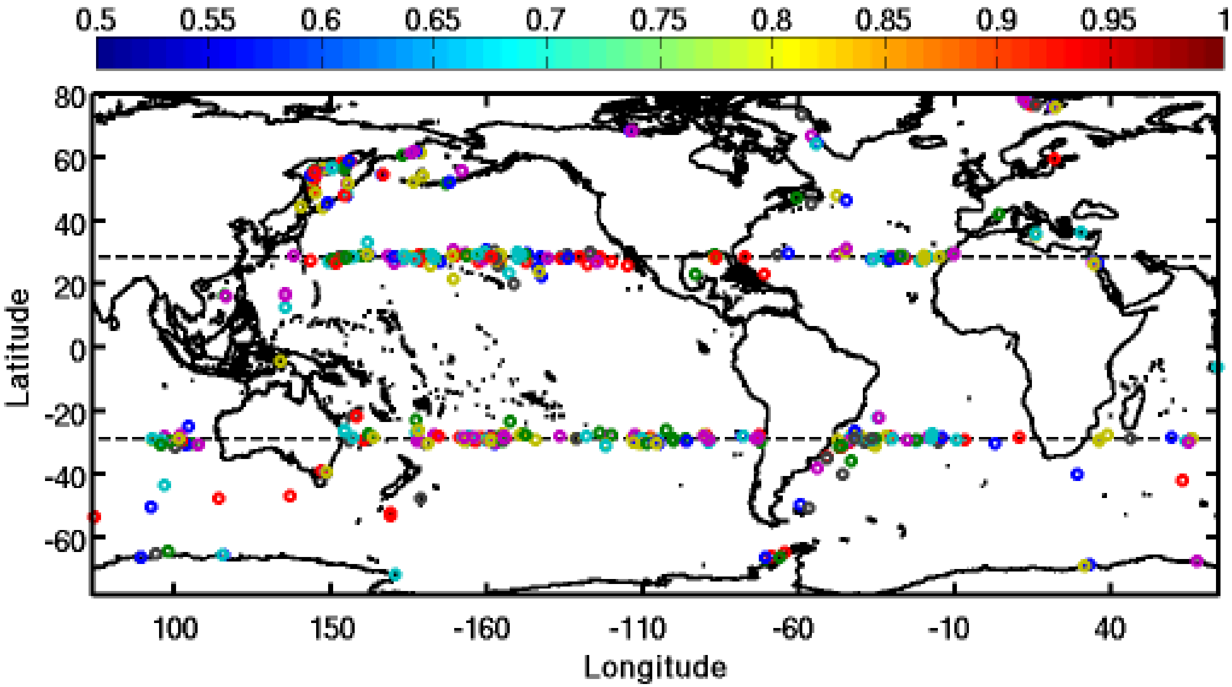
BIG CHALLENGE:

- How to separate subharmonics $\&$ Wind-generated NIO...

INDIRECT APPROACH:

- Invoke 'energy ratio', R, since...
- R discriminates weak/strong 'energy' signals

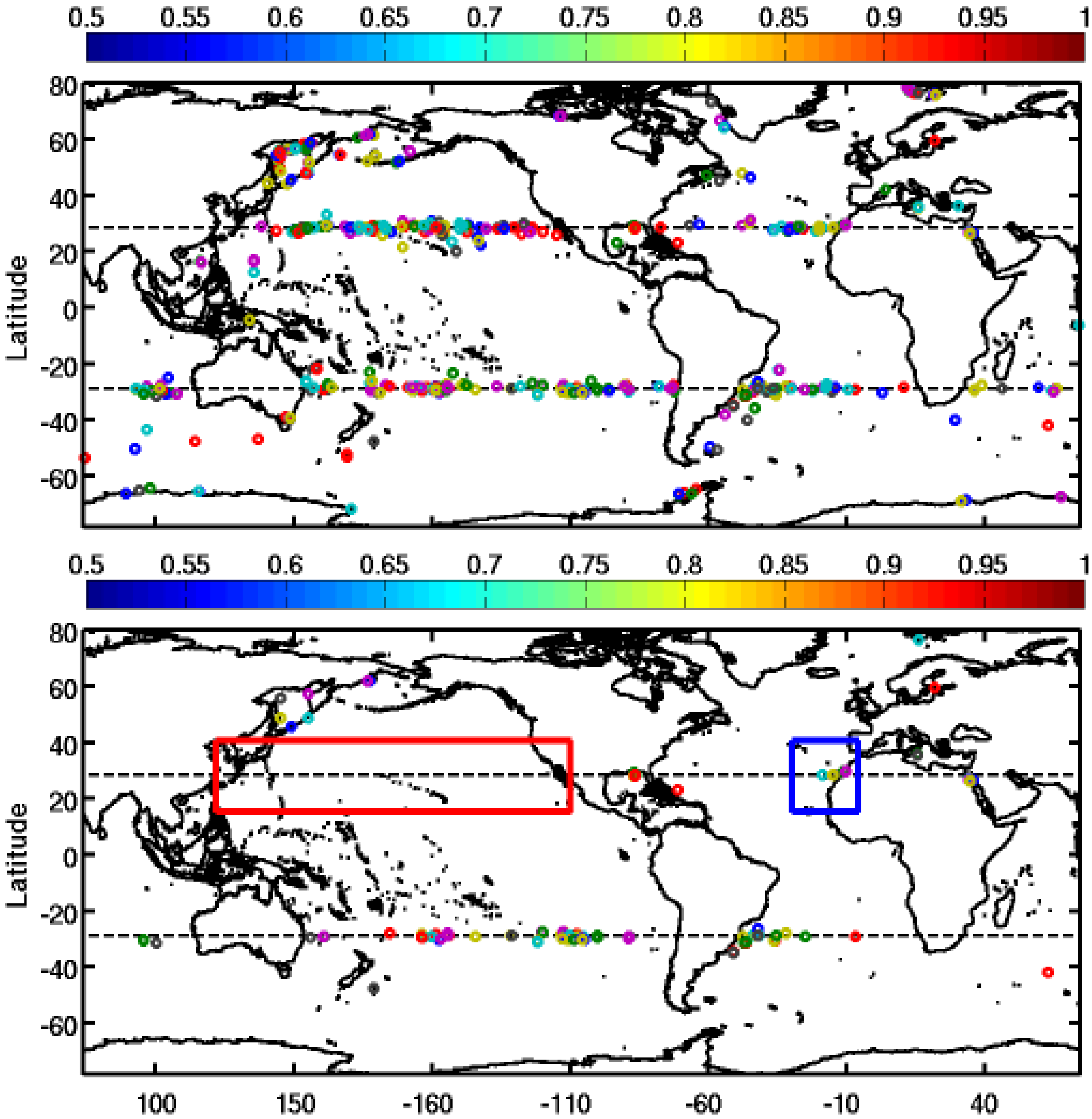
Fraction of Power in Subharmonics



Bicoherence...

Eliminate: $R < 1\,\%$

Fraction of Power in Subharmonics



Bicoherence...

Eliminate: $R < 1\,\%$

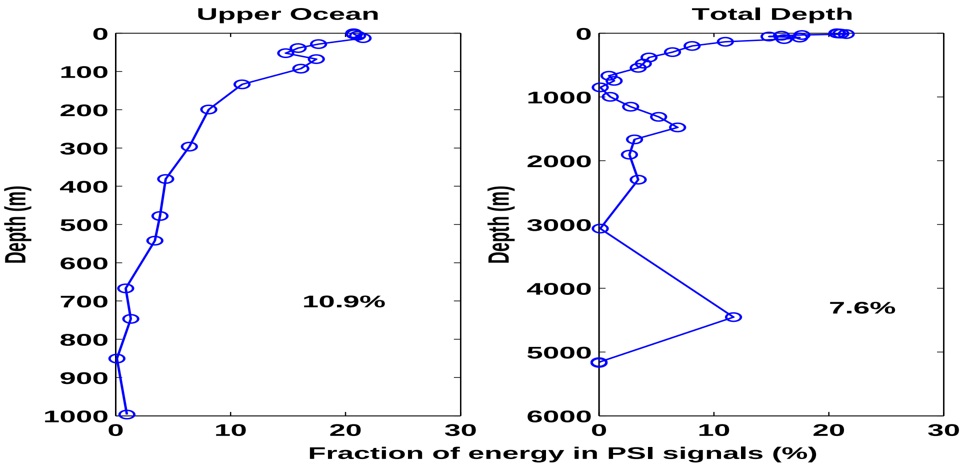
Bicoherence...

Eliminate: $R < 7\,\%$

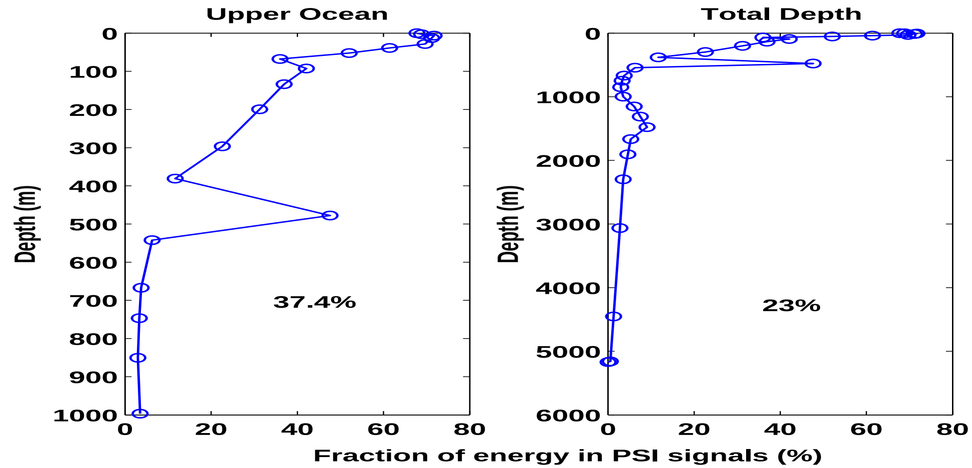
Fraction of power in...
N. Pacific layer 14 is...
at most $7\,\%$

Fraction of Power in Subharmonics

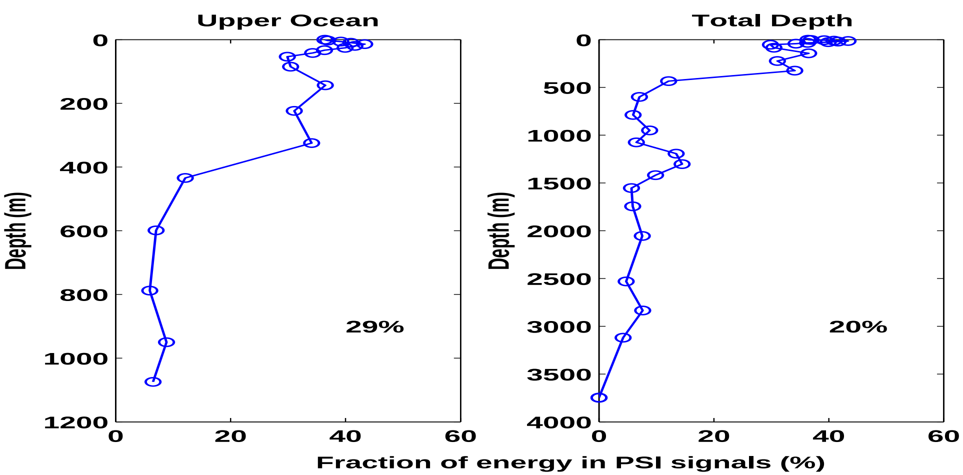
North Pacific



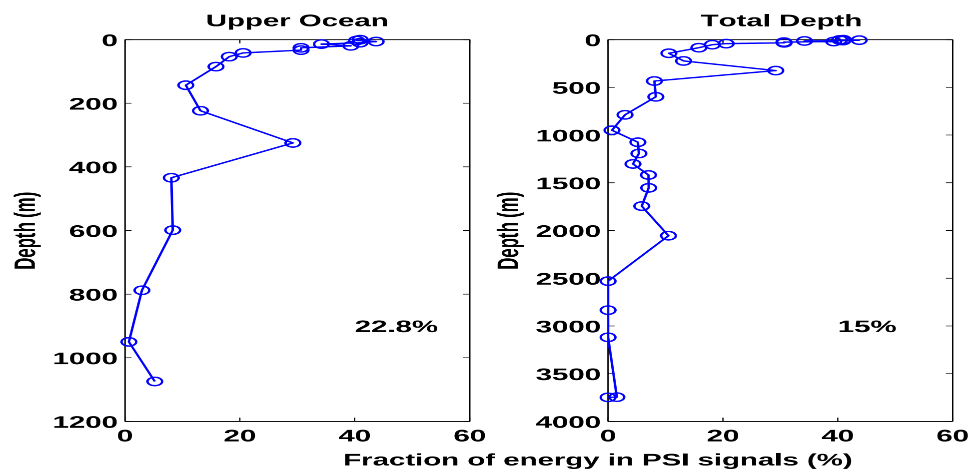
North Atlantic



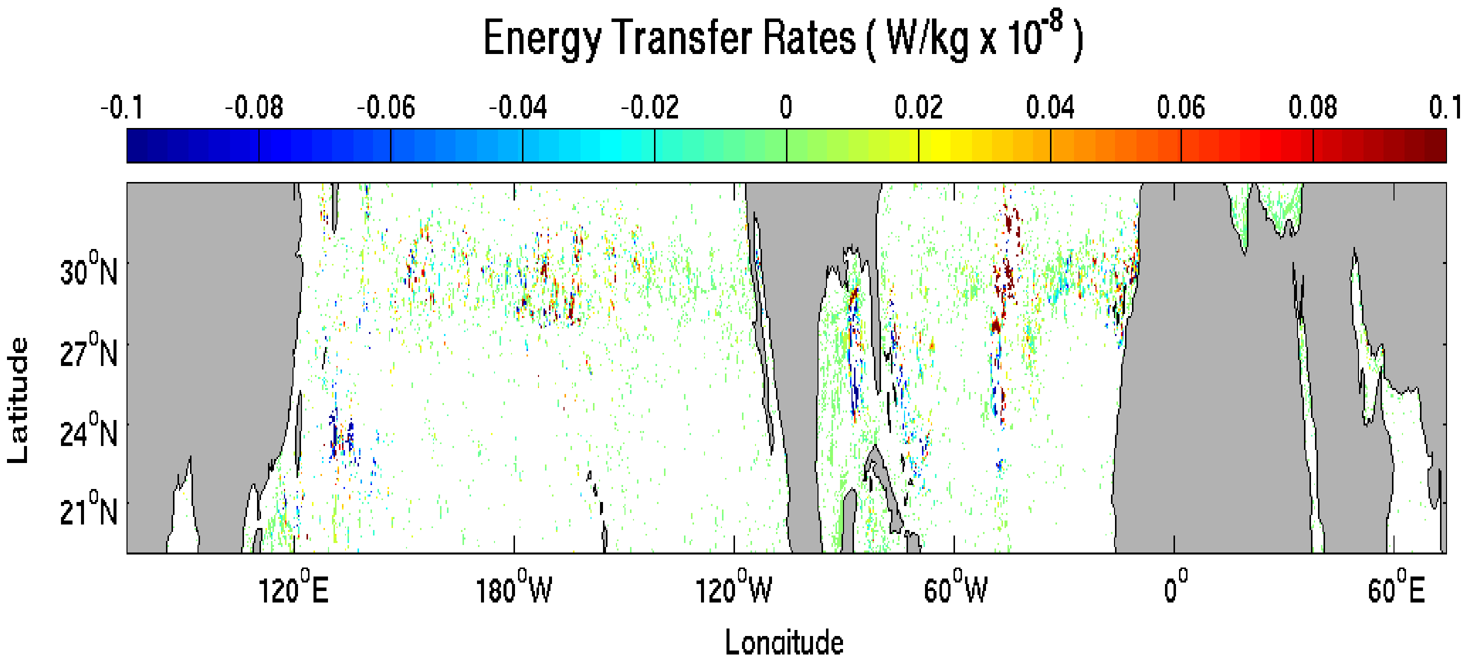
South Pacific



South Atlantic



Rates of Energy Transfer: North

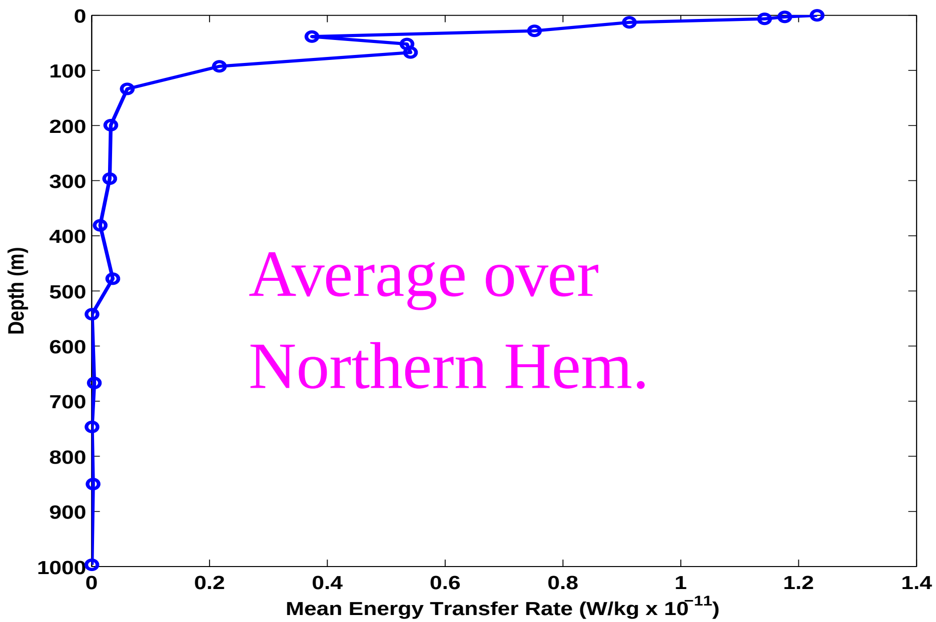


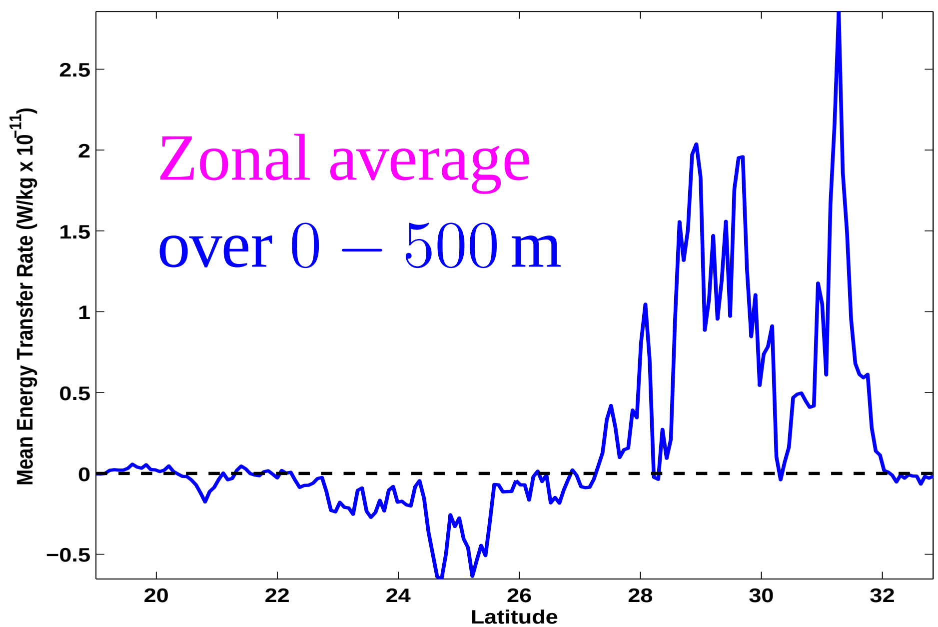
Highest transfer rates around Atlantic

formulas:

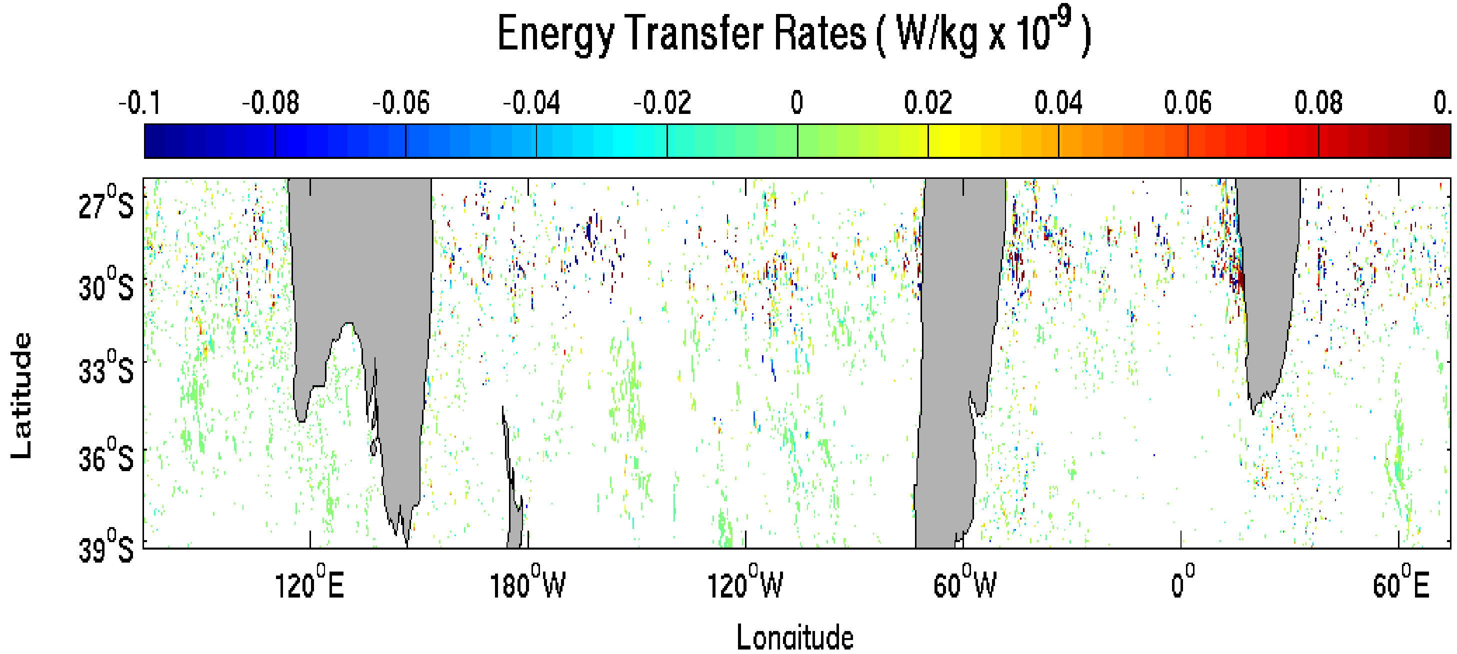
MacKinnon et. al (2012)

Sun & Pinkel (2013)



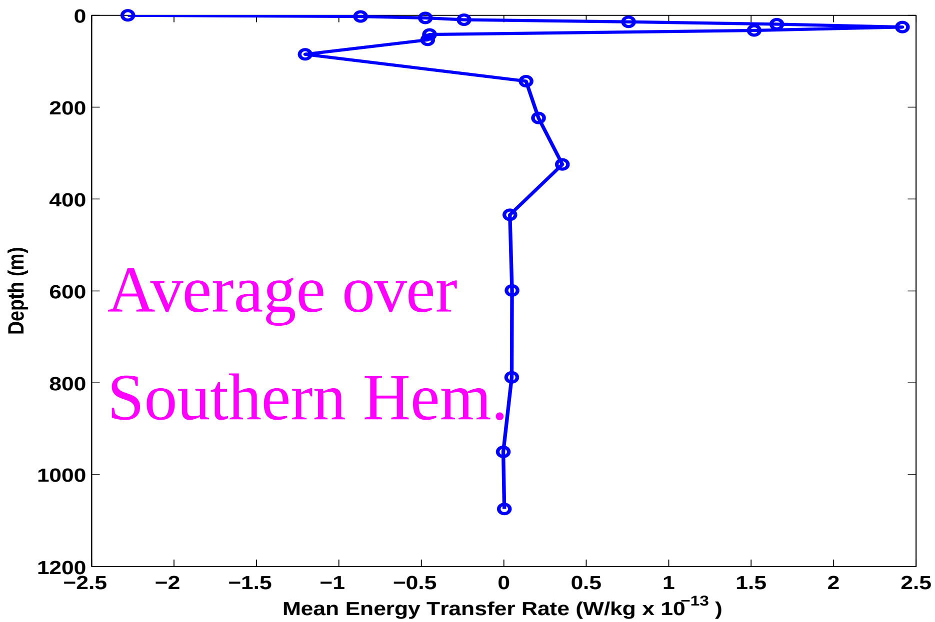


Rates of Energy Transfer: South

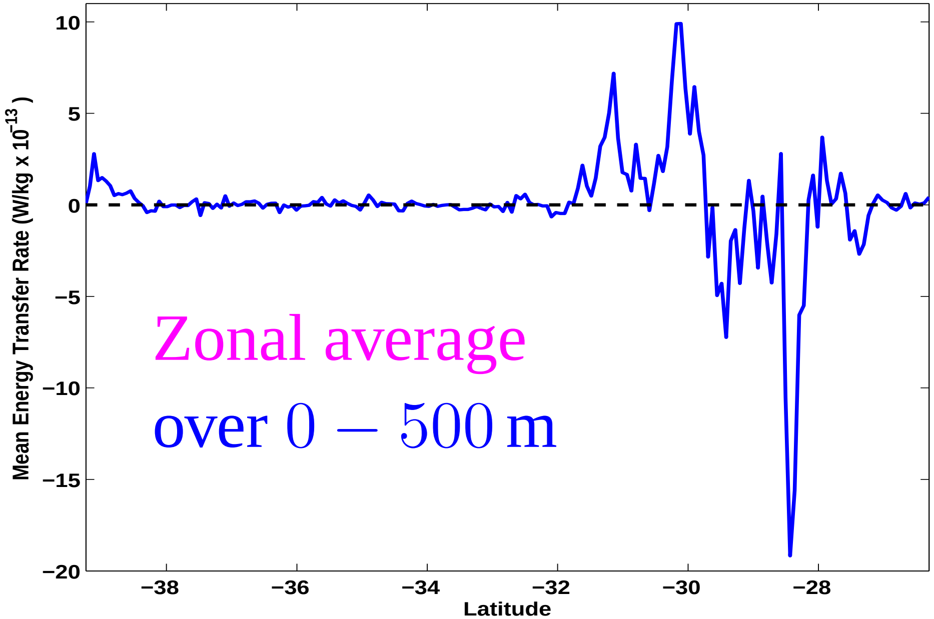


Smaller rates of transfer

Transfer is almost equal in both directions



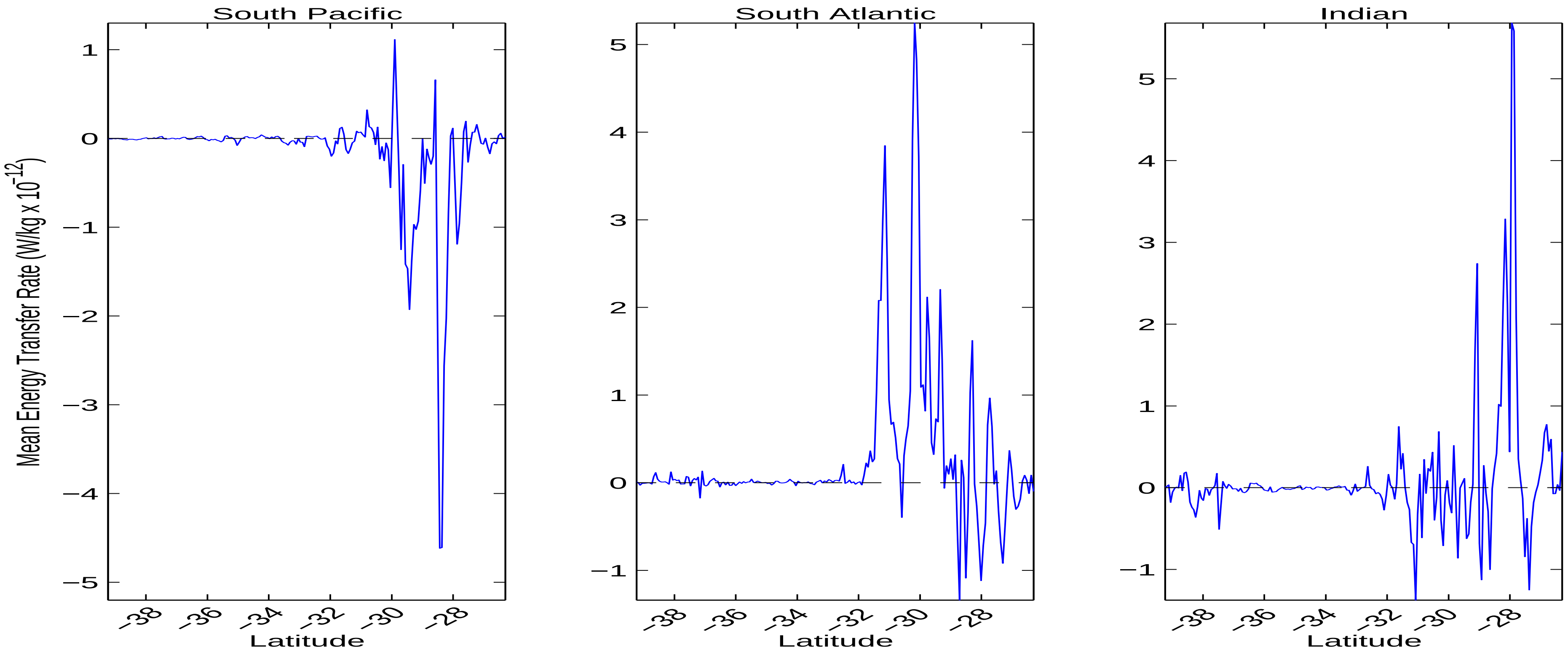
Average over Southern Hem.



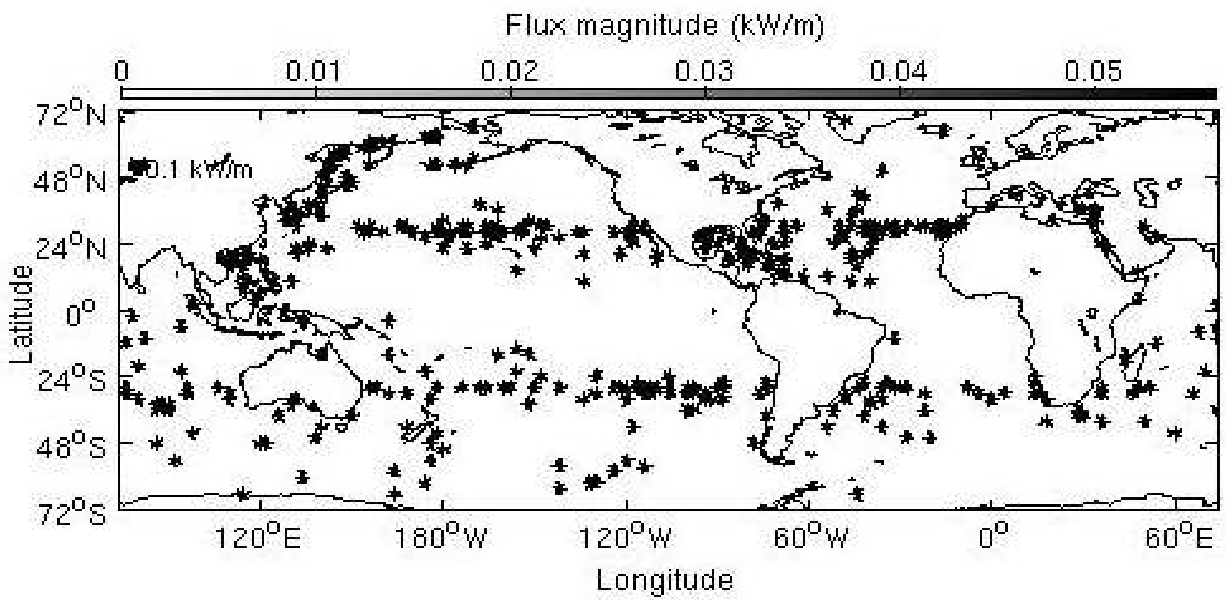
Zonal average over $0 - 500\,\mathrm{m}$

Rate of Energy Transfer: South

Most negative transfers occur in the South Pacific.



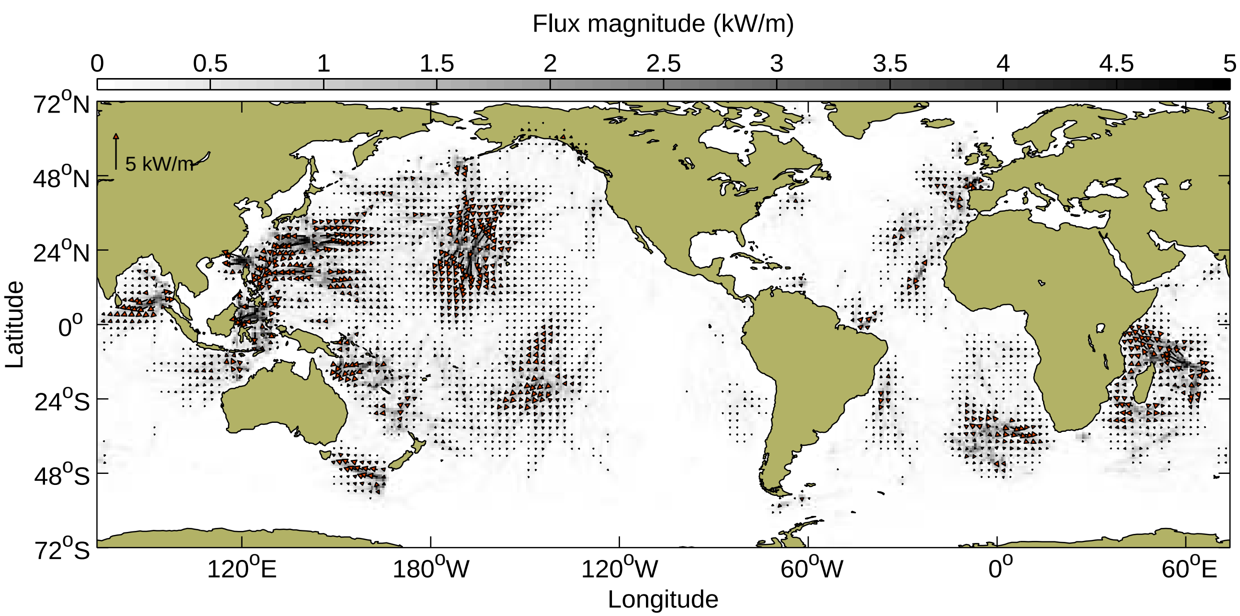
Subharmonic/Tidal Energy Flux



Vertically-integrated subharmonic energy flux

[depth: 0 – 1000 m]

Vectors not to scale!!

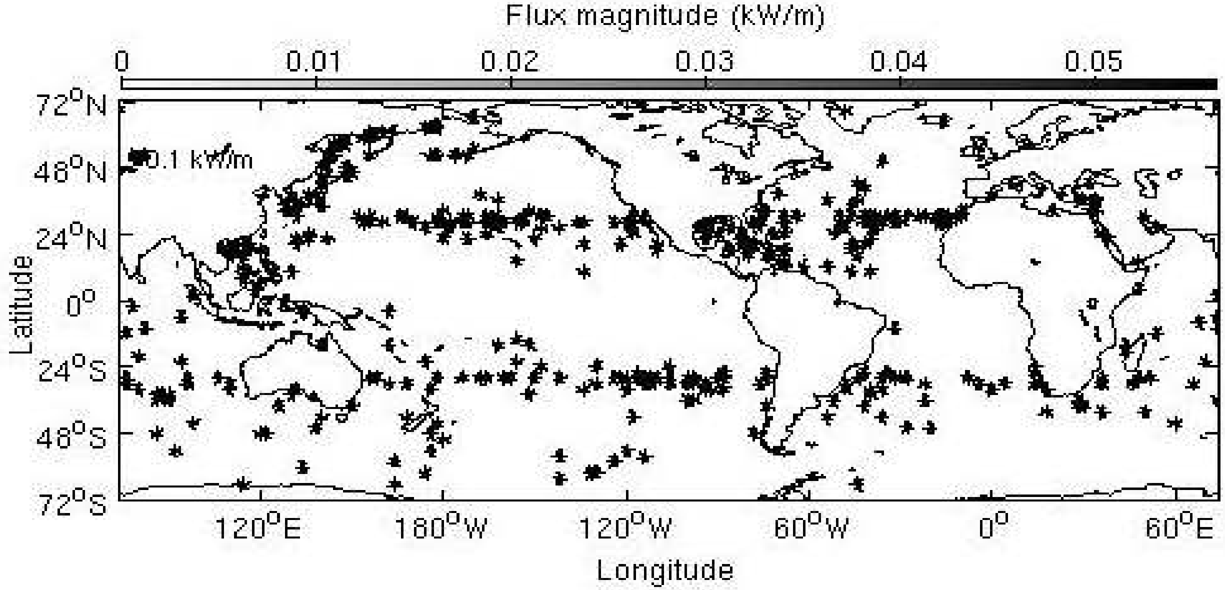


Baroclinic

Semi-diurnal fluxes

Sample of vectors plotted

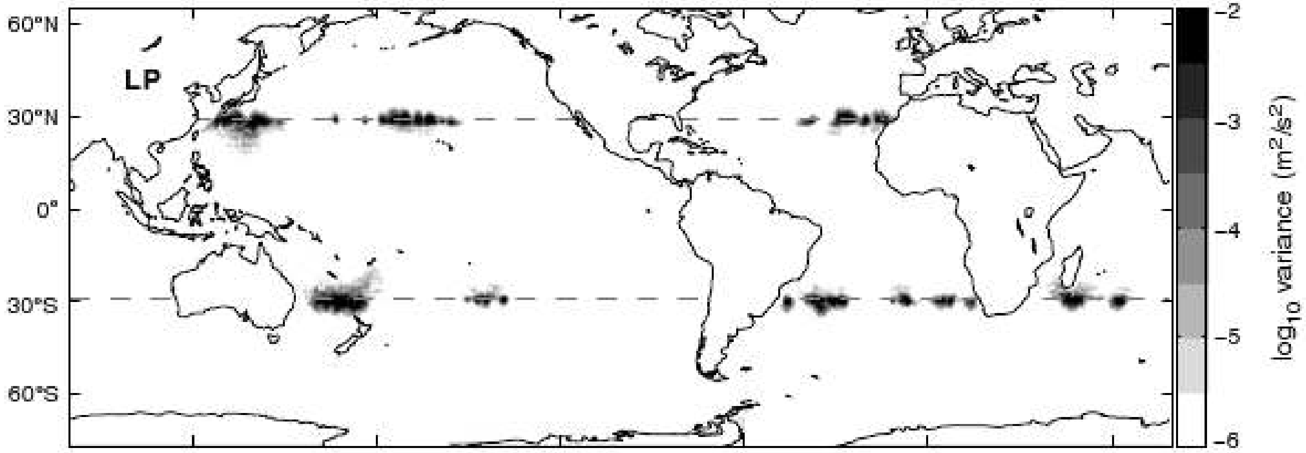
Compare to Simmons (2008)



Vertically-integrated subharmonic energy flux

[depth: 0 – 1000 m]

Vectors not to scale!!



3. Geographic distribution of upper ocean baroclinic velocity variance [var(u') + var(v')] averaged over days 160–170. The upper panel shows the averaged total variance ("Full"). The bottom panel shows frequencies lower than M_2 ("LP"). The SHTL is indicated by the dashed lines along ±28.8° e bottom panel.

Simmons (2008)
Subharmonic velocity variance

Summary & Conclusions

Global distribution of PSI in high resolution HYCOM show

- PSI around critial latitude (CL) of tides via bicoherence calculations
- Intense semi-diurnal PSI in upper ocean [Hazewinkel& Winters 2011]
- Intensity falls within 3° of CL [Furuichi et al. (2005)]
- Estimates of fraction of energy in subharmonics $O(8-23\,\%)$ [consistent with IWAP estimates around Hawai'i: $O(10-20\,\%)$]
- Positive rates of energy transfer in Northern Hemisphere...

THANK YOU

Extra stuff Extra stuff Extra stuff

Rate of Energy Transfer

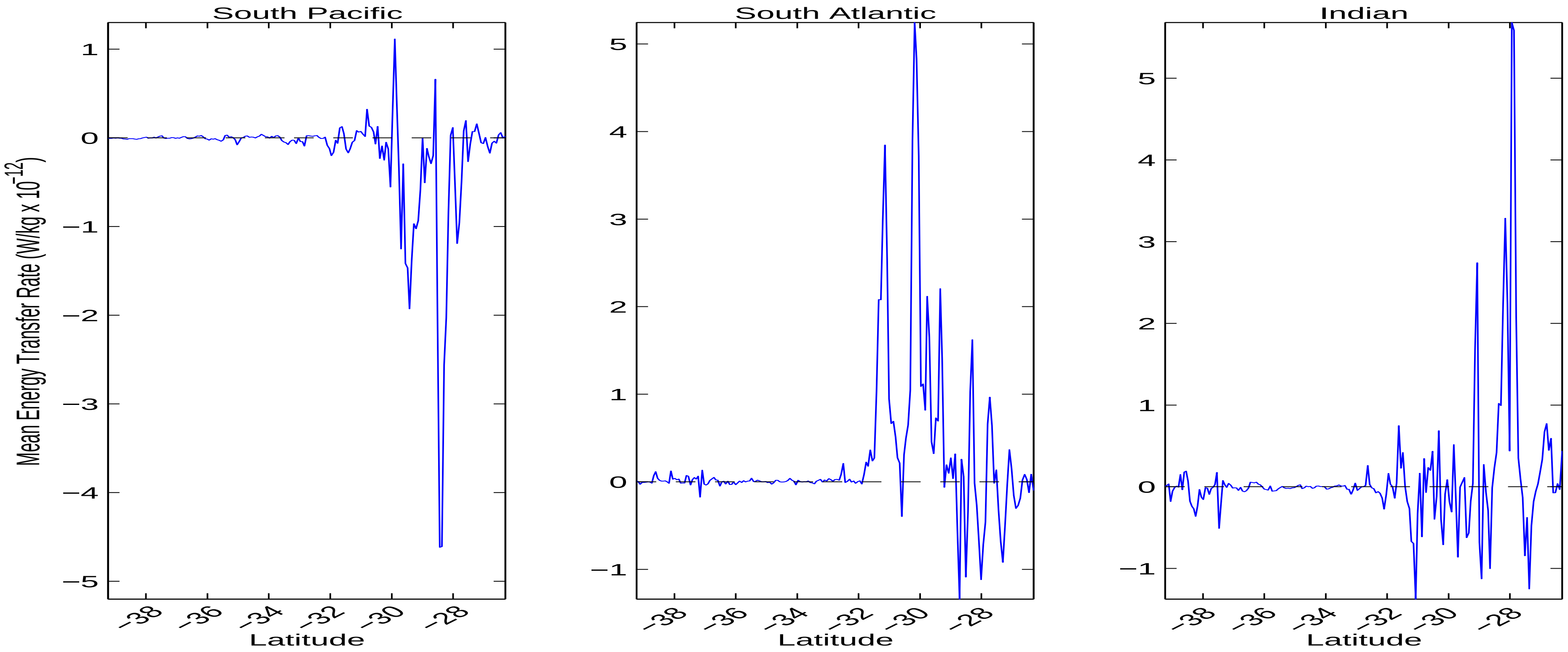
- Employ recent formulas (MacKinnon et. al. 2012, Sun & Pinkel, 2013).
- Example: $(u_1, u_2) \rightarrow$ daughter waves, $u_3 \rightarrow$ tide.

$$\begin{aligned} E_t &= \frac{3}{2} ik\hat{u}_1\hat{u}_2\hat{u}_3^* + c.c. \\ \mathrm{E}[E_t] &= \frac{3}{2} ikB(\omega_1, \omega_2) + c.c. \end{aligned}$$

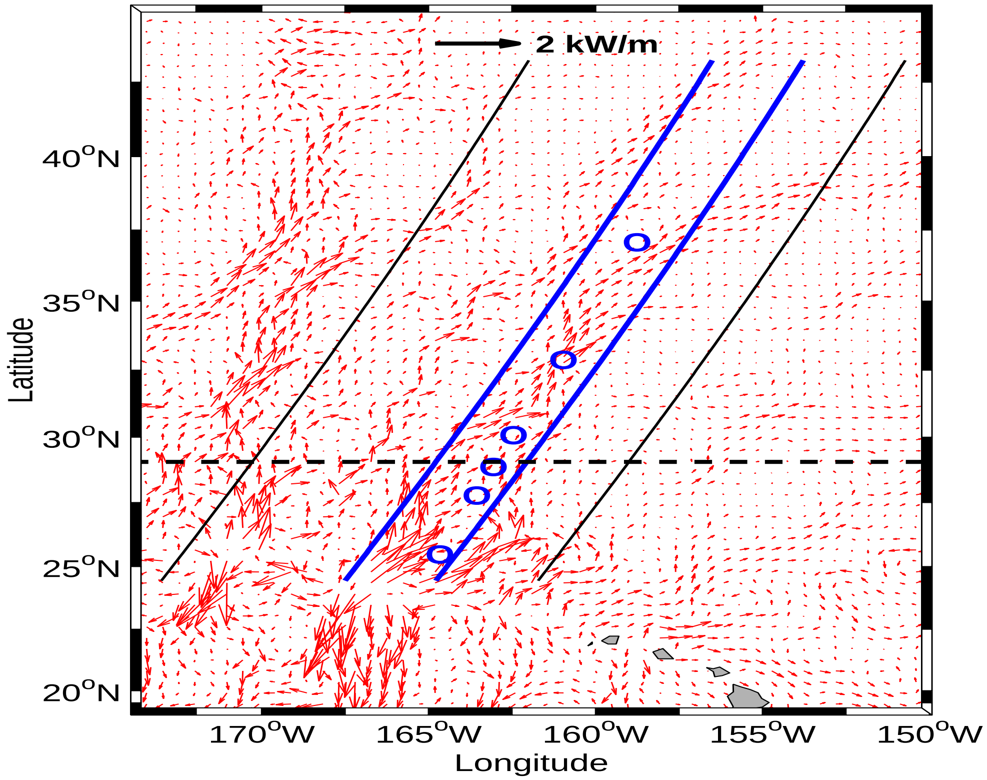
$k =$ horizontal wavenumber.

$E_t > 0 \Longrightarrow$ Transfer from tides $\rightarrow$ subharmonics
$E_t < 0 \Longrightarrow$ Transfer from tides $\leftarrow$ subharmonics

Rate of Energy Transfer

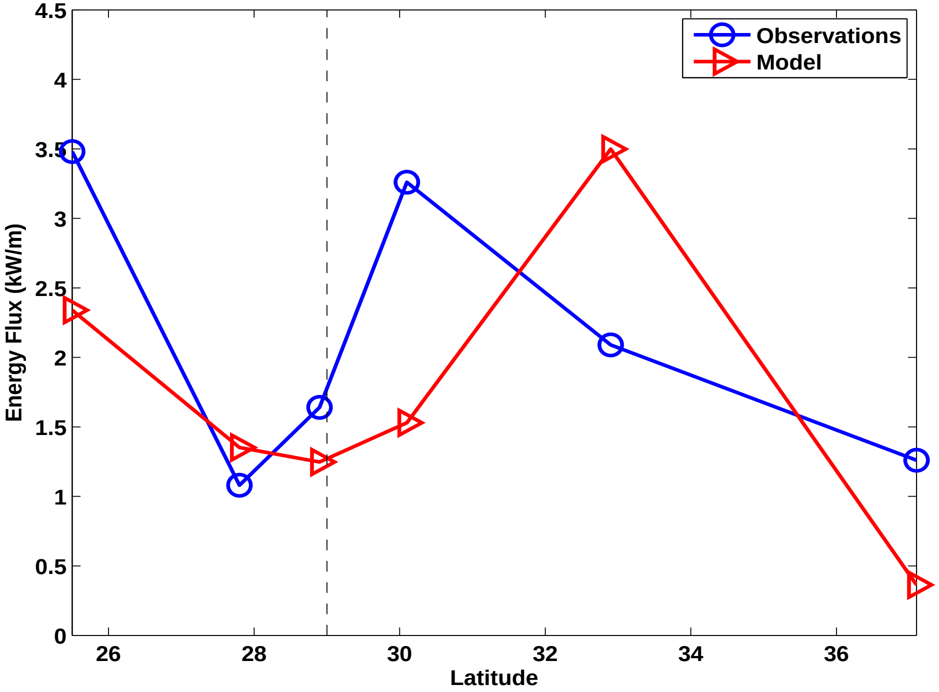


Energy Flux Comparison

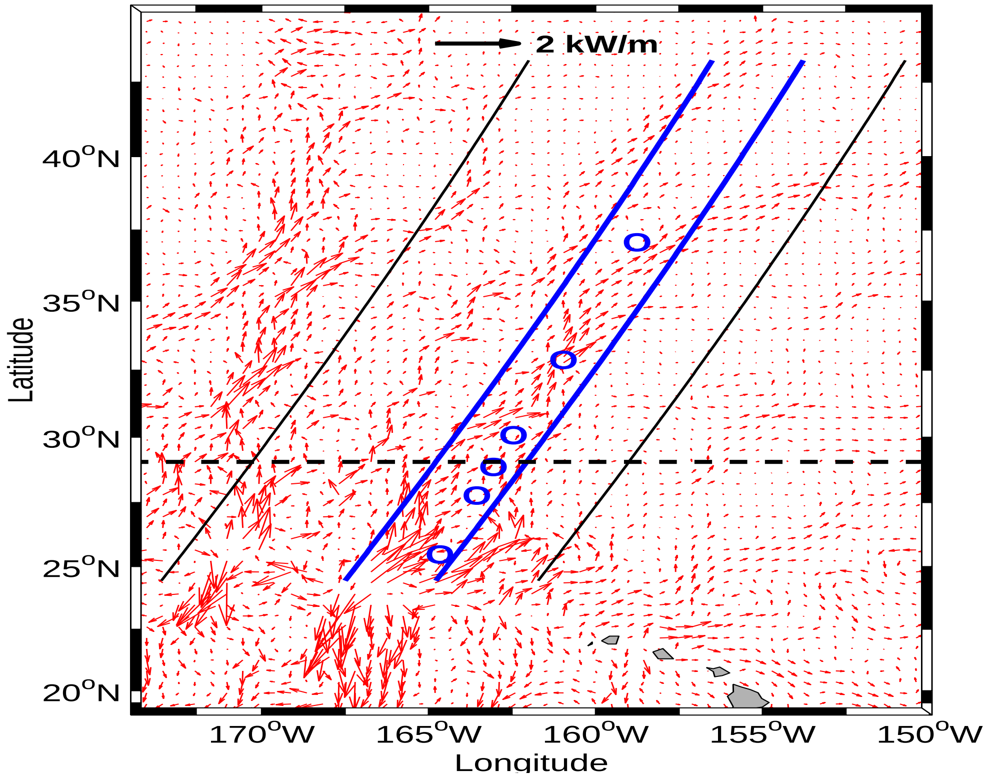


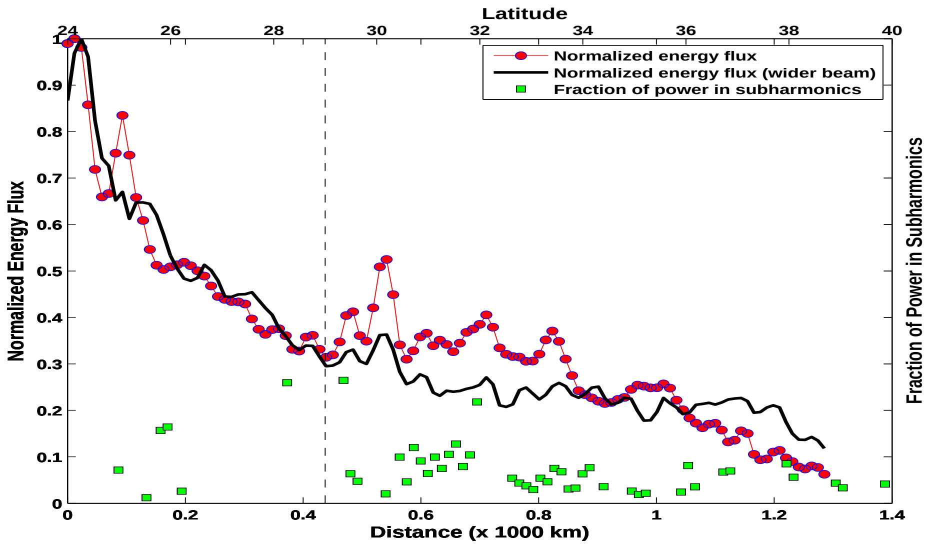
M_2 Energy fluxes (HYCOM)

IWAP observations (blue circles)



Energy Flux Comparison





M_2 Energy fluxes (HYCOM)

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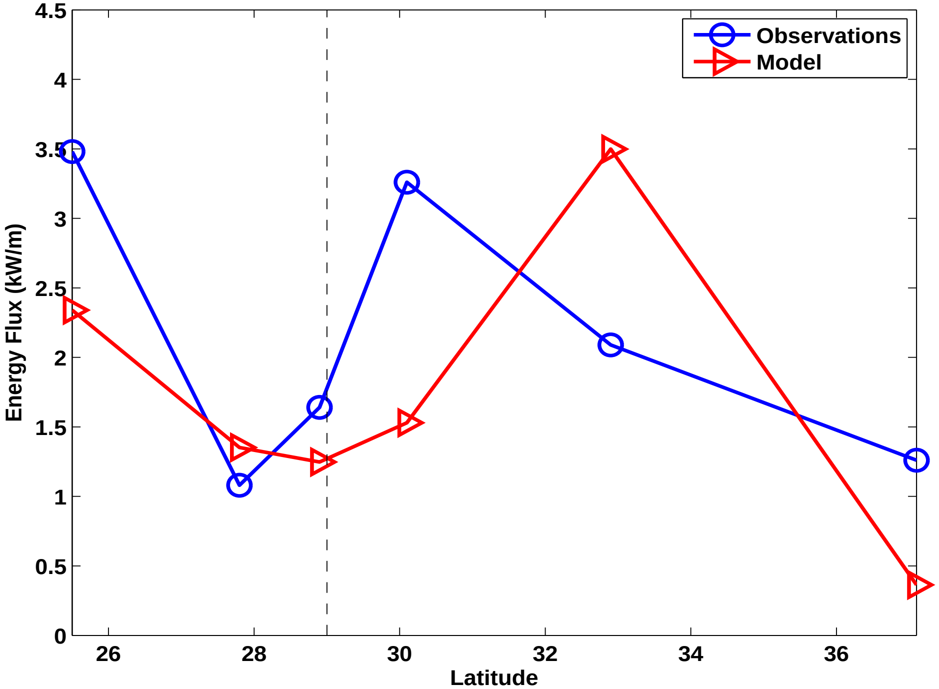


Bispectrum Approach

$x(t)$ = zero-mean signal
$X(\omega)$ = complex Fourier transform of $x(t)$

Bispectrum (Kim & Powers, 1979)

$$B(\omega_1, \omega_2) = E[X(\omega_1)X(\omega_2)X^*_{\omega_1+\omega_2}]$$

Implications: $B(\omega_1, \omega_1) = 0$ unless

- ω_1, ω_2 and $\omega_3 = \omega_1 + \omega_2$ are present
- phase coherence is present in waves

Bicoherence: normalized bispectrum

Quantitative measure is bicoherence:

$$b^2(k,l) = \frac{|B(\omega_1,\omega_2)|^2}{E[|X_{\omega_1}X_{\omega_2}|^2]E[|X_{\omega_1+\omega_2}|^2]}$$

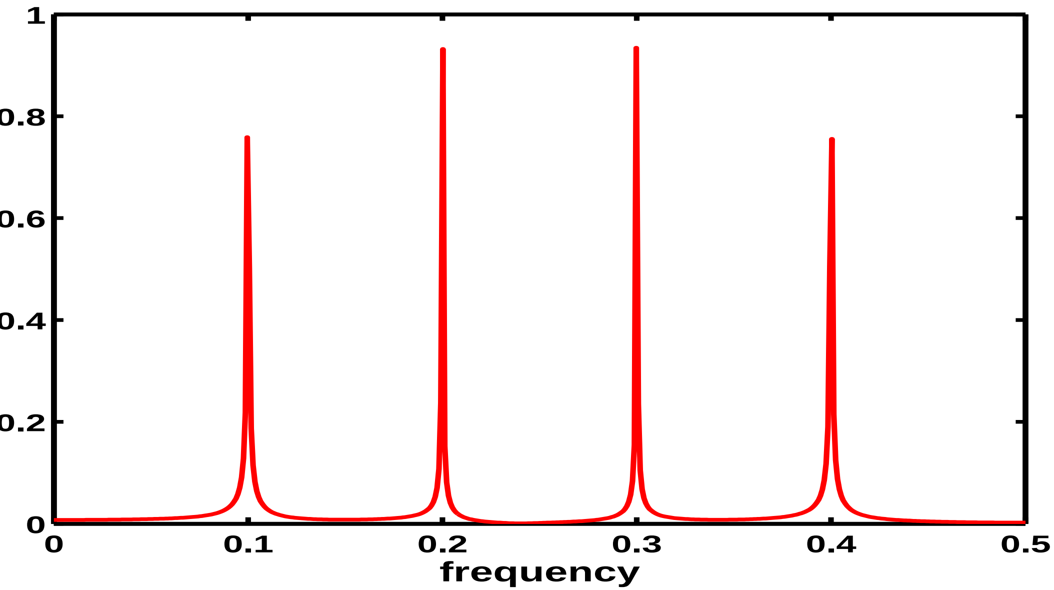
with $0 \leq b(\omega_1,\omega_2) \leq 1$.

Significance levels of b (Elgar & Guza, 1988):

$$95\% \equiv \sqrt{6/n_{dof}};\ 99\% \equiv \sqrt{9/n_{dof}}$$

where n_{dof} = number of degrees of freedom (tricky issue, Carter & Gregg 2006, Sun & Pinkel 2010, MacKinnon et al. 2011)

Illustrative Example

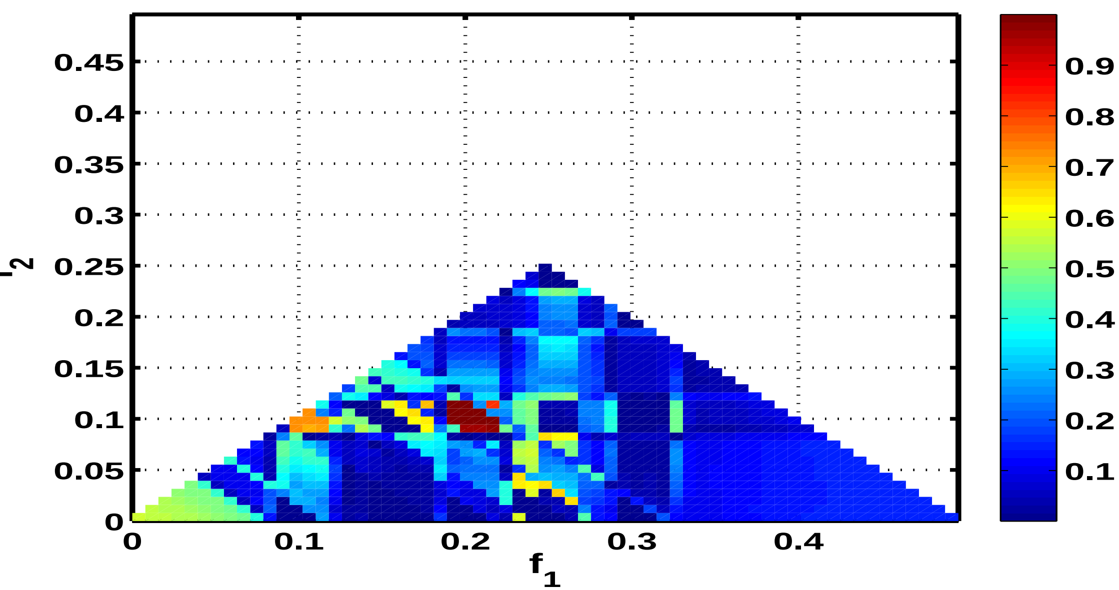


$$y = \cos(2\pi f_1 t + \phi_1) + \cos(2\pi f_2 t + \phi_2) + \cos(2\pi f_3 t + \phi_3) + \cos(2\pi f_4 t + \phi_4)$$

$$f_1 + f_2 = f_3$$

$$\phi_1 + \phi_2 = \phi_3$$

$$f_1 = 0.2,\ f_2 = 0.1,\ f_4 = 0.4$$



What is this R?

Kim & Powers (1979) show that:

$$b^2(\omega_1,\omega_2)E[|X_{\omega_3}|^2] = |A_{1,2}|^2 E[|X_{\omega_1}X_{\omega_2}|^2] \cdots\cdots (1)$$

represents the power at ω_3 due to the coupling at ω_1 and ω_2 where $A_{1,2}$ is the interaction coefficient:

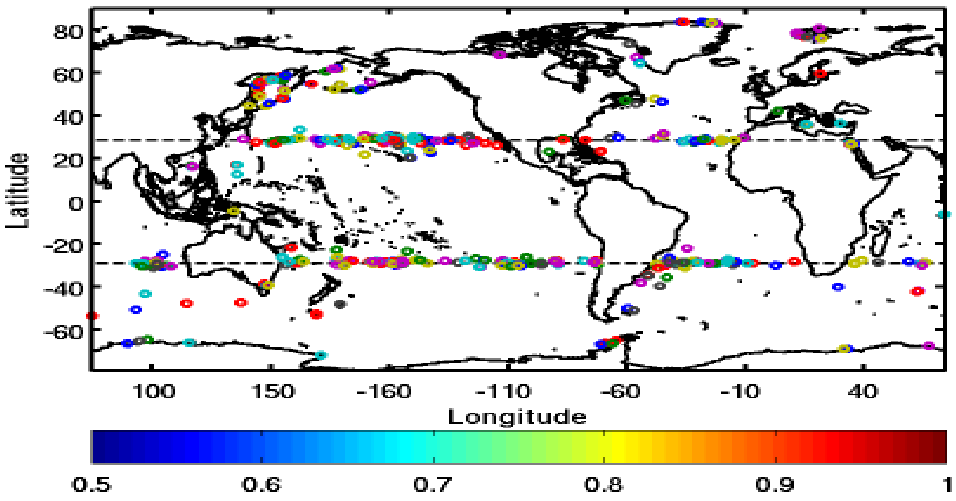
$$X_{\omega_3} = A_{1,2}X_{\omega_1}X_{\omega_2}, \quad A_{1,2} = B^*(\omega_1,\omega_2)/E[|X_{\omega_1}X_{\omega_2}|^2]$$

Take square root of (1) and get

$$R = \frac{b}{b + |A_{1,2}|}$$

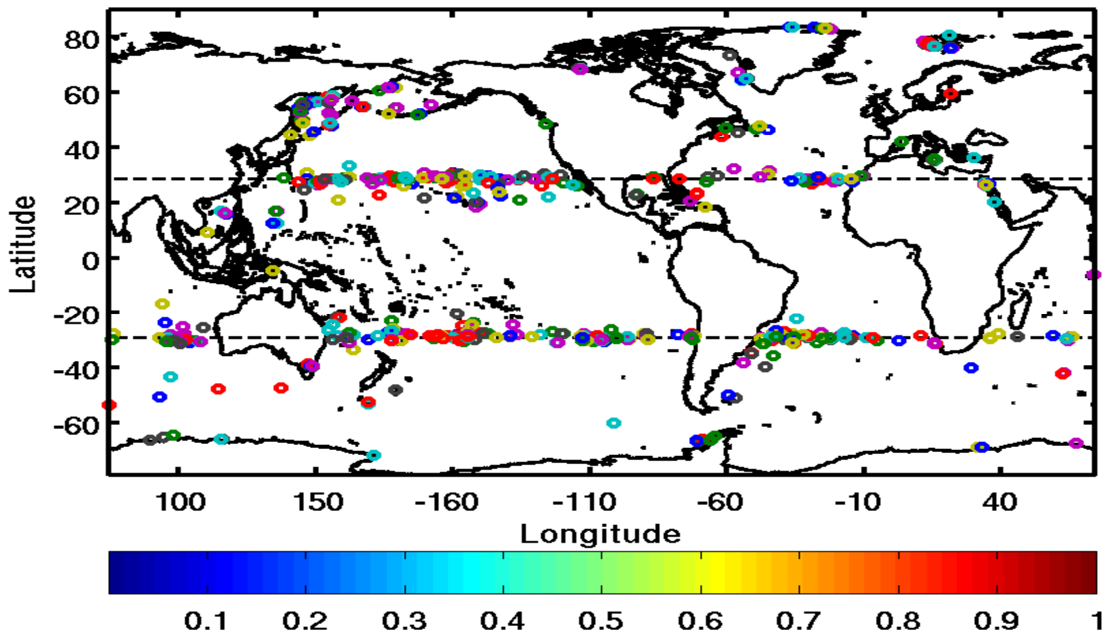
Similar cleaner global maps may be obtained by plotting $A_i = 1/|A_{1,2}|$

Bicoherence vs Interaction Coefficient



$b(\omega_1, \omega_2)$

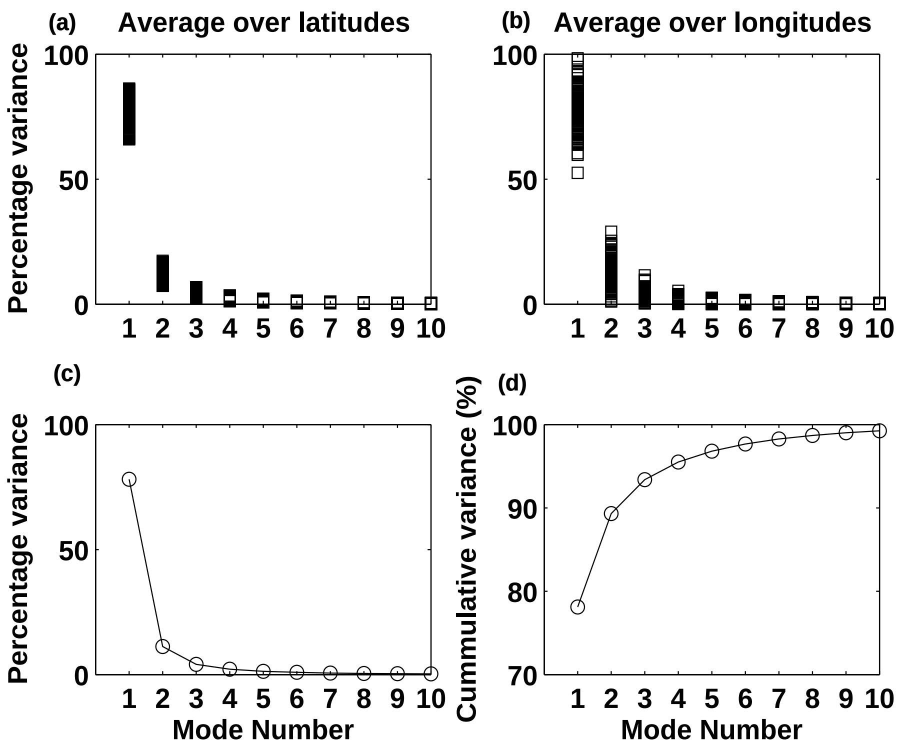
Eliminate: $R < 1\,\%$



$A_i = 1/|A_{1,2}|$

Eliminate: $A_i < 1.4\,\%$

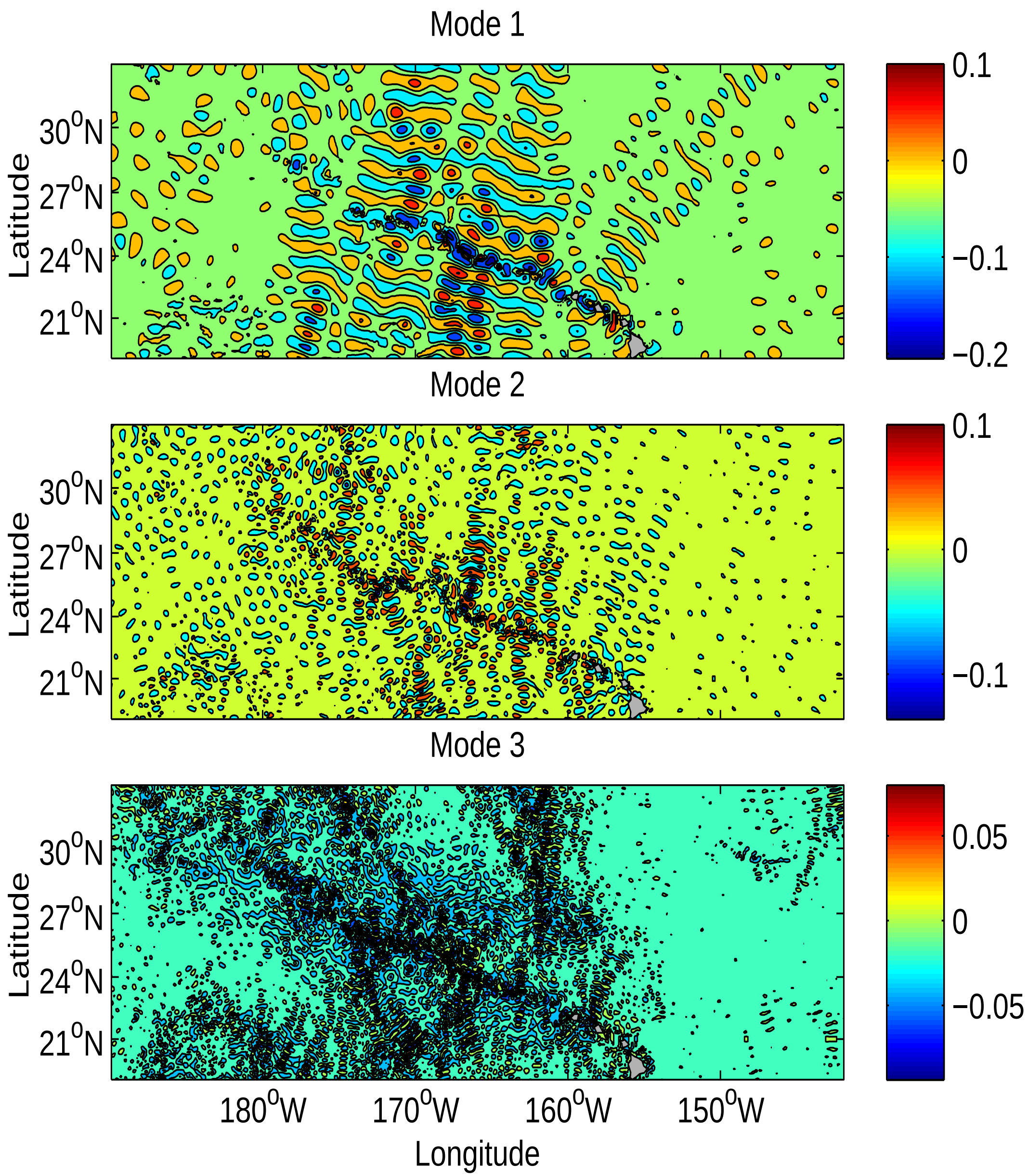
Number of vertical modes resolved



First 4 modes explain....

95 % of variance

Spatial modal structures



Stratification vs Bicoherence