Turbulence examined in the frequency-wavenumber domain*

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* Arbic et al. JPO 2012; Arbic et al. in review; Morten et al. papers in preparation

Collaborators

- University of Michigan Brian Arbic, Charlie Doering
- MIT Glenn Flierl
- University of Brest, and The University of Texas at Austin Robert Scott

Outline of talk

Part I–Motivation (research led by Brian Arbic)

• Frequency-wavenumber analysis:

–Idealized Quasi-geostrophic (QG) turbulence model.

–High-resolution ocean general circulation model (HYCOM)*.

–AVISO gridded satellite altimeter data.

*We used NLOM in Arbic et al. (2012)

Part II-Research led by Andrew Morten

• Derivation and interpretation of spectral transfers used above.

• Frequency-domain analysis in two-dimensional turbulence.

• Theoretical prediction for frequency spectra and spectral transfers due to the effects of "sweeping."

–moving beyond a zeroth order approximation.

Motivation: Intrinsic oceanic variability

• Interested in quantifying the contributions of intrinsic nonlinearities in oceanic dynamics to oceanic frequency spectra.

• Penduff et al. 2011: Interannual SSH variance in ocean models **with** interannual atmospheric forcing

is comparable to

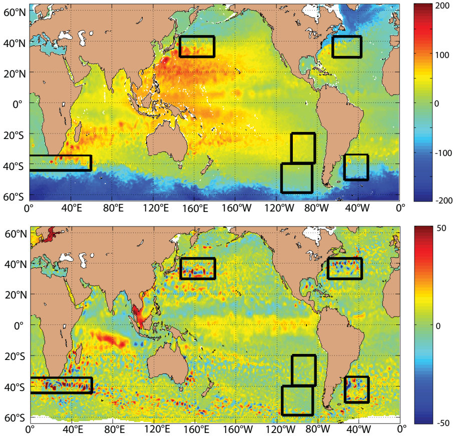
variance in high resolution (**eddying**) ocean models **with no** interannual atmospheric forcing.

• Might this eddy-driven low-frequency variability be connected to the well-known inverse cascade to low wavenumbers (e.g. Fjortoft 1953)?

• A separate motivation is simply that transfers in mixed $\omega - k$ space provide a useful diagnostic.

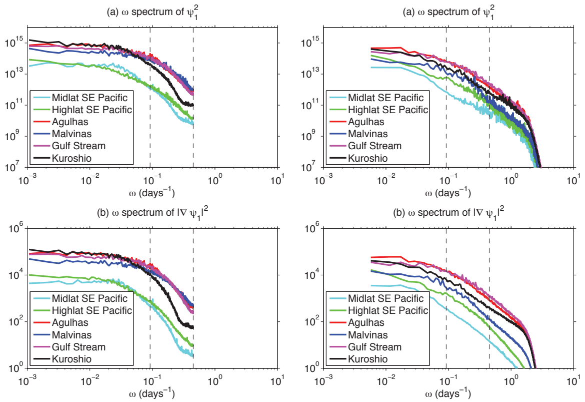
Oceanic analysis regions

- Regions used to analyze AVISO gridded altimeter data and HYCOM output.
- Shown against snapshots of SSH (cm) from HYCOM and SSH anomaly (cm) from AVISO.



Frequency spectra of surface streamfunction variance ψ_1^2 and kinetic energy $|\nabla\psi_1|^2$ (Arbic et al. 2012 JPO)

AVISO NLOM

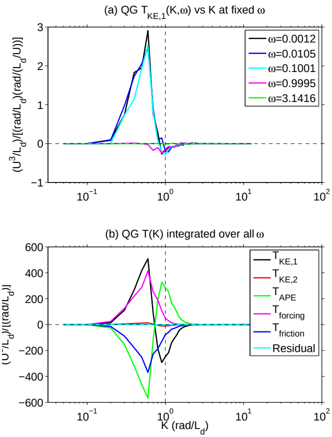


- Flat spectra at low frequencies as in previous studies (e.g. Richman et al. 1977, Wunsch 2009, 2010)

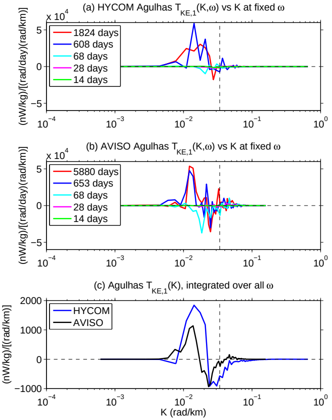
Spectral transfers of upper layer kinetic energy versus wavenumber at fixed frequency

Spectral transfers vs. k

QG model



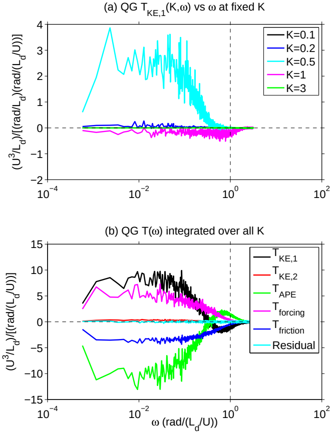
Agulhas HYCOM and AVISO



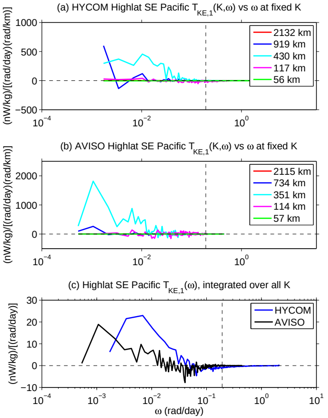
Spectral transfers of upper layer kinetic energy versus frequency at fixed wavenumber

Spectral transfers vs. ω

QG model



Highlat SE Pac HYCOM & AVISO



Derivation of spatial transfers

- Reminder of derivation of energy and enstrophy spatial transfers

$$\frac{\partial}{\partial t}\nabla^2\psi + J = F + D \tag{1}$$

$$\Downarrow$$

$$-\frac{\partial}{\partial t}k^2\tilde{\psi} + \tilde{J} = \tilde{F} + \tilde{D} \tag{2}$$

$$\Downarrow$$

Energy: $$\frac{\partial}{\partial t}k^2\frac{|\tilde{\psi}(\vec{k},t)|^2}{2} - \mathrm{Re}(\tilde{\psi}^*\tilde{J}) = -\mathrm{Re}(\tilde{\psi}^*\tilde{F}) - \mathrm{Re}(\tilde{\psi}^*\tilde{D}) \tag{3}$$

and

Enstrophy: $$\frac{\partial}{\partial t}k^4\frac{|\tilde{\psi}|^2}{2} - \mathrm{Re}(k^2\tilde{\psi}^*\tilde{J}) = -\mathrm{Re}(k^2\tilde{\psi}^*\tilde{F}) - \mathrm{Re}(k^2\tilde{\psi}^*\tilde{D}) \tag{4}$$

Time-frequency analysis needed for temporal transfers

Choose either:

- Short-time (moving window) Fourier transform:

$$\hat{f}(\tau,\omega;T) := \int_{\tau-T/2}^{\tau+T/2} \sigma(\frac{t-\tau}{T})[f(t) - f_{trend}(t,\tau;T)]e^{-i\omega t}dt \quad (5)$$

$$\implies \widehat{\frac{df}{dt}}(\tau,\omega) = i\omega\hat{f} + \frac{\partial}{\partial\tau}\hat{f}(\tau,\omega) \quad (6)$$

- Wavelet:

$$\hat{f}(\tau,\omega) := \int_{-\infty}^{\infty} f(t)|\omega|^{1/2}W((t-\tau)\omega)dt \quad (7)$$

$$\implies \widehat{\frac{df}{dt}}(\tau,\omega) = \frac{\partial}{\partial\tau}\hat{f}(\tau,\omega) \quad (8)$$

Derivation of temporal transfers

- Derivation of energy and enstrophy temporal transfers:

$$-\widehat{\frac{\partial}{\partial t}k^2\tilde{\psi}} + \hat{\tilde{J}} = \hat{\tilde{F}} + \hat{\tilde{D}} \tag{9}$$

$$\Downarrow$$

$$-\frac{\partial}{\partial \tau}k^2\hat{\tilde{\psi}} + \underbrace{(-i\omega k^2\hat{\tilde{\psi}})}_{\text{only for STFT}} + \hat{\tilde{J}} = \hat{\tilde{F}} + \hat{\tilde{D}} \tag{10}$$

$$\Downarrow$$

$$\text{Energy:}\quad \frac{d}{d\tau}k^2\frac{|\hat{\tilde{\psi}}(\vec{k},\tau,\omega)|^2}{2} - \mathrm{Re}(\hat{\tilde{\psi}}^*\hat{\tilde{J}}) = -\mathrm{Re}(\hat{\tilde{\psi}}^*\hat{\tilde{F}}) - \mathrm{Re}(\hat{\tilde{\psi}}^*\hat{\tilde{D}}) \tag{11}$$

and

$$\text{Enstrophy:}\quad \frac{d}{d\tau}k^4\frac{|\hat{\tilde{\psi}}|^2}{2} - \mathrm{Re}(k^2\hat{\tilde{\psi}}^*\hat{\tilde{J}}) = -\mathrm{Re}(k^2\hat{\tilde{\psi}}^*\hat{\tilde{F}}) - \mathrm{Re}(k^2\hat{\tilde{\psi}}^*\hat{\tilde{D}}) \tag{12}$$

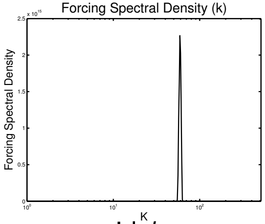
Model choices

- 2D Navier-Stokes equation (modified)

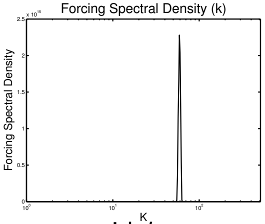
$$\frac{\partial}{\partial t}\nabla^2\psi + J(\psi, \nabla^2\psi) = F - \nu_0\nabla^2\psi + D_{hyper} + D_{hypo} \qquad (13)$$

- streamfunction, ψ
- jacobian, $J(\cdot, \cdot)$
- Ekman drag, $\nu_0 = 0$, for this talk.
- exponential wavenumber filter (hyper-)viscosity, D_{hyper}
- " " filter (hypo-)viscosity, D_{hypo}
- forcing, F, chosen to be localized about wavenumber k_F and frequency ω_F.

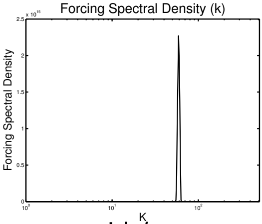
Three different forcing frequencies



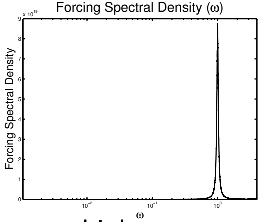
mid k_F



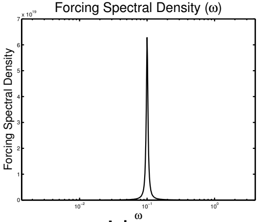
mid k_F



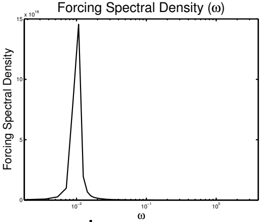
mid k_F



high ω_F



mid ω_F



low ω_F

Model choices: spectrally localized forcing

In order to allow for statistically homogeneous isotropic turbulence...

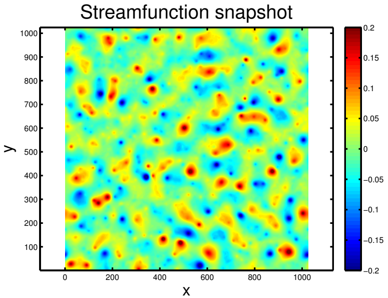
Start with Maltrud-Vallis (stochastic) forcing,

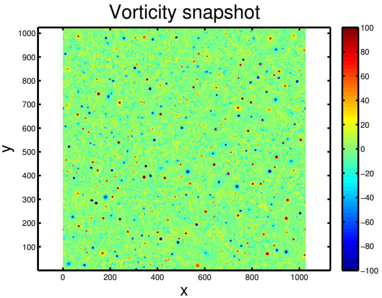
$$\tilde{F}_{MV}(\vec{k}, t_n) = f_0\sqrt{1 - c^2}e^{i\phi_n} + c\tilde{F}_{MV}(t_{n-1}), \tag{14}$$

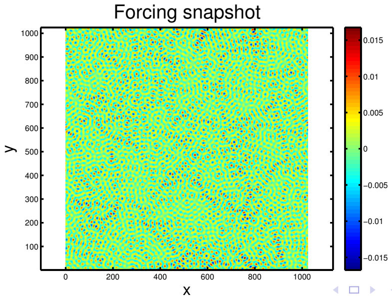
Shift the power spectrum peak to ω_F and $-\omega_F$

$$\tilde{F}(\vec{k}, t_n) = e^{i\omega_F t_n}\tilde{F}^{+}_{MV}(\vec{k}, t_n) + e^{-i\omega_F t_n}\tilde{F}^{-}_{MV}(\vec{k}, t_n) \tag{15}$$

Typical $\psi(\vec{x}, t)$ and $\nabla^2\psi$ snapshots; and forcing

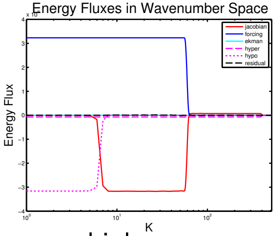




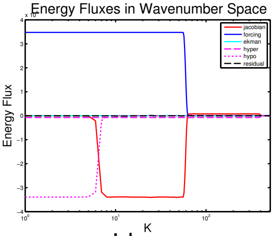


Energy Fluxes in k-space (three different forcing periods)

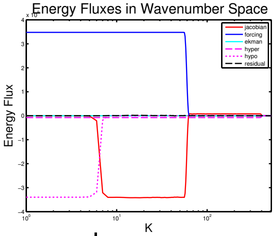
Energy Fluxes (nonlinear term in red) in k-space



high ω_F



mid ω_F

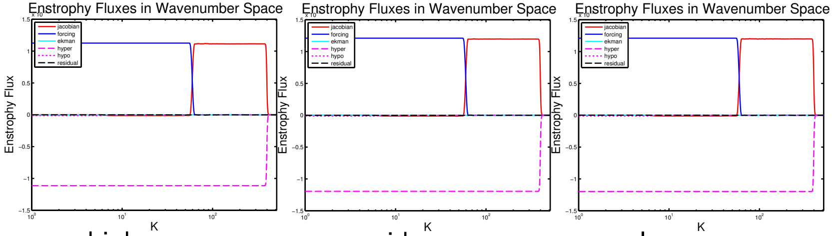


low ω_F

- Clear inertial ranges in k-space.
- Similar results for all three forcing periods.
  - slightly more injection of energy for longer forcing periods.

Enstrophy Fluxes in k-space (three forcing periods)

Enstrophy Fluxes (nonlinear term in red) in k-space

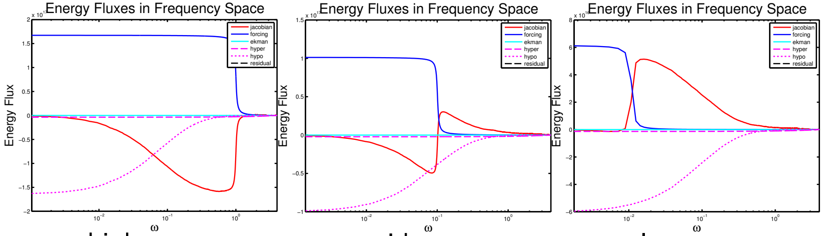


high ω_F mid ω_F low ω_F

- Clear inertial ranges in k-space.
- Similar results for all three forcing periods.
  - slightly more injection of enstrophy for long forcing periods.

Energy Fluxes in ω-space (three forcing periods)

Energy Fluxes (nonlinear term in red) in ω-space



high ω_F mid ω_F low ω_F

- Not really an "inertial range" in ω-space.
- Direction of "cascade" depends on forcing frequency.
- Probably these transfers are determined by sweeping (by v_{rms}),
  - more complicated than a simple Taylor's hypothesis.

Why can the spectral transfers proceed in either direction

Local conservation within triads in k-space (Kraichnan 1967)

$$\begin{aligned} 0 &= T(k,p,q) + T(p,q,k) + T(q,k,p), \\ 0 &= k^2 T(k,p,q) + p^2 T(p,q,k) + q^2 T(q,k,p), \end{aligned} \tag{16}$$

generalizes to

$$\begin{aligned} 0 &= T(k,p,q;\omega_k,\omega_p,\omega_q) + T(p,q,k;\omega_p,\omega_q,\omega_k) + T(), \\ 0 &= k^2 T(k,p,q;\omega_k,\omega_p,\omega_q) + p^2 T(p,q,k;\omega_p,\omega_q,\omega_k) + q^2 T(), \end{aligned} \tag{17}$$

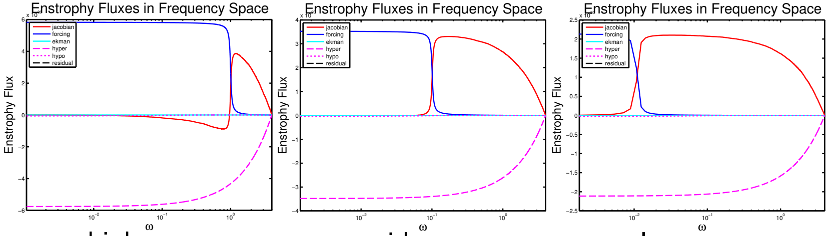
which, by integrating over wavenumbers, gives

$$\begin{aligned} 0 &= Q_1(\omega_k,\omega_p,\omega_q) + Q_1(\omega_p,\omega_q,\omega_k) + Q_1(), \\ 0 &= Q_2(\omega_k,\omega_p,\omega_q) + Q_2(\omega_p,\omega_q,\omega_k) + Q_2(), \end{aligned} \tag{18}$$

where Q_1 and Q_2 are energy and enstrophy transfers, respectively, within frequency triads.

Enstrophy Fluxes in ω-space (three forcing periods)

Enstrophy Fluxes (nonlinear term in red) in ω-space

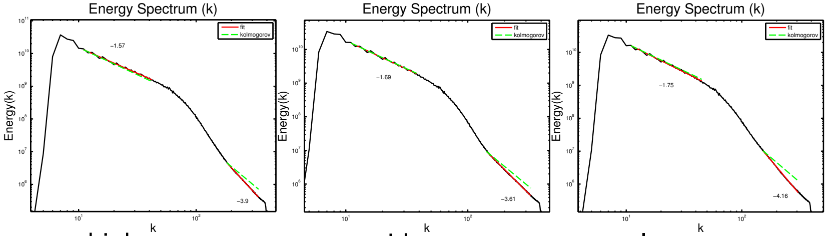


high ω_F mid ω_F low ω_F

- Not really an "inertial range" in ω-space, except for small ω_F.
- Direction of "cascade" depends on forcing frequency.
- Probably these transfers are determined by sweeping (by v_{rms}),
  - more complicated than a simple Taylor's hypothesis.

Energy Spectra in k-space (three forcing periods)

Energy spectra in k-space

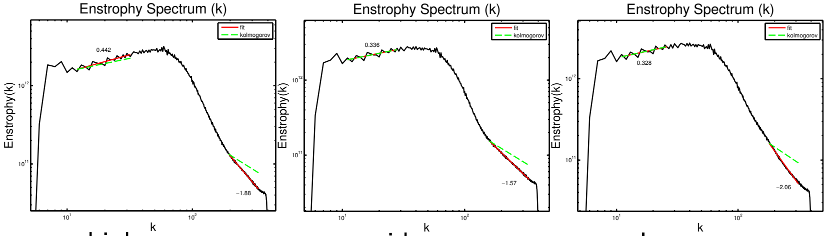


high ω_F mid ω_F low ω_F

- Spectral slopes reasonably close to Kraichnan's scaling in k-space.
- $(-5/3$ and -3, or more like -4)

Enstrophy Spectra in k-space (three forcing periods)

Enstrophy spectra in k-space



high ω_F mid ω_F low ω_F

- Spectral slopes reasonably close to Kraichnan's scaling in k-space.

Taylor's hypothesis and sweeping

An overview of Taylor's hypothesis and sweeping.

- Taylor's hypothesis:

(strong mean flow) + (spatial structure)

$$\implies \text{(Eulerian frequency spectra).}$$

- sweeping hypothesis:

(zero mean flow) + (strong large-scale v_{rms}) + (spatial structure)

$$\implies \text{(Eulerian frequency spectra).}$$

Taylor's hypothesis and sweeping

• sweeping hypothesis:
(zero mean flow) + (strong large-scale v_{rms}) + (spatial structure)
$\Longrightarrow$ (Eulerian frequency spectra).

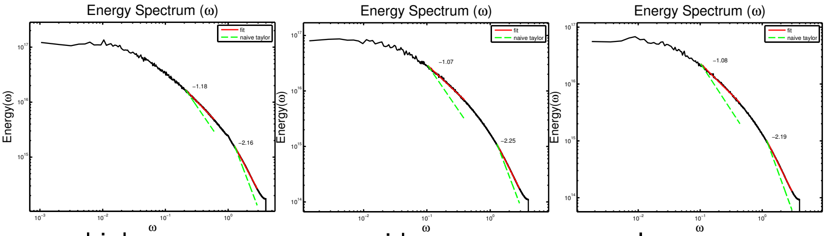
– sweeping is typically treated similarly to a strong mean flow, but with velocity v_{rms}. (Tennekes 1975)

– or, as an ensemble of time-independent sweeping velocities. (Chen et. al. 1989)

– but for a general time-dependent sweeping, one needs to be careful. (Morten et al., in preparation)

Energy Spectra in ω-space (three forcing periods)

Energy spectra in ω-space

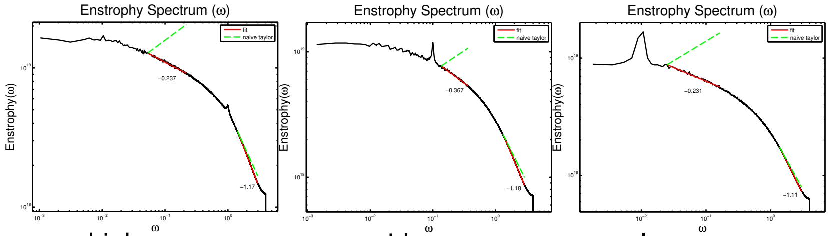


high ω_F mid ω_F low ω_F

- Taylor's hypothesis and "sweeping" are not quite right; slopes not equal to those in wavenumber spectrum.
- Slopes at low frequencies near -1; higher frequencies near -2.

Enstrophy Spectra in ω-space (three forcing periods)

Enstrophy spectra in ω-space



high ω_F mid ω_F low ω_F

- (Naive) Taylor's hypothesis or "sweeping" give incorrectly signed slope.
- Direct cascade enstrophy spectra give best match (-1 slope).
- (Correct) Taylor's hypothesis or "sweeping" give negative slope in ω-spectra for a positive sloping k-spectra (see next two slides).

Taylor and Sweeping done right

Summary of some analytical results:

- Taylor done right:

(mean velocity v_0) and spectra $E(k) \propto k^p$ (between k_{min} and k_{max})

$$\implies E(\omega) \approx \omega^p, \text{ for } p < 0$$

$$\implies E(\omega) \approx \omega^{-p(\omega/v_0 k_{max})^p} \approx \omega^{-0}, \text{ for } p > 0$$

–Why? Because in 2D or 3D, $\vec{v_0}$ and $\vec{k}$ need not be parallel, so every mode k contributes to arbitrarily low ω in $E(\omega)$.

- sweeping done right:

(zero mean velocity $v_0 = 0$) and spectra $E(k) \propto k^p$ (as above)

$$\implies E(\omega) \sim \omega^p + \Omega^2\omega^{p-2} + O(\omega^{p-4}), \text{ for } p < 0,$$

as an asymptotic series in the limit $\omega \to \infty$, and where

Ω depends on v_{rms}, $\langle \vec{v}'(t) \cdot \vec{v}'(t) \rangle$, k_{min} and k_{max}, and note that Ω could be quite large.

Summary

• We derived and gave an interpretation for spectral transfers in the wavenumber-frequency domain.

• We do see a transfer of energy to lower frequencies due to nonlinearity in some parts of the "ocean" (as measured by AVISO or modelled by HYCOM or a two-layer QG model).

• In the case of 2D turbulence, the direction of the transfer is largely determined by the effects of sweeping.

• A more rigorous analysis of a time-dependent sweeping is needed to understand slopes of frequency spectra, especially when the cascade is narrow in wavenumber.

Summary of part I

• Spectral transfers T and fluxes Π display a tendency for nonlinearities in idealized geostrophic turbulence models to drive energy into low frequencies as well as low wavenumbers.

• Low wavenumber energy associated with low frequencies and vice versa, but not in a simple way.

• Realistic OGCM's also display a general tendency for nonlinearities to drive energy into lower frequencies, though not as simply or consistently as QG in turbulence models. AVISO gridded altimeter data does only in some of the regions examined; possibly because it is a highly filtered product.