A Mechanical Energy Budget and Evaluation of an Eddying Global Ocean Model with a Wave Drag Parameterization

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Outline

1. **Introduction**
   - Motivation and what wave drag is
   - The model and observations for comparison
2. **Putting wave drag into an ocean model**
   - Wave drag scheme choices
3. **Energy budget**
   - Mechanical energy budget from the continuity and momentum equations
4. **Model evaluation**
   - Taylor diagrams of all five diagnostics

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- Wave drag boosts vertical diffusivity (e.g., *St. Laurent et al.*, 2002) and improves all considered tidal constituent amplitudes (e.g., *Jayne and St. Laurent*, 2001) in barotropic tidal models
- Offline estimates suggest wave drag dissipates energy at $0.2 - 0.49$ TW in abyssal hill regions (e.g., *Nikurashin and Ferrari*, 2011; *Scott et al.*, 2011)

A history of topographic wave drag improving models (contd...)

Our goals

- How do we insert wave drag into an eddying global ocean model (**without tides**)?

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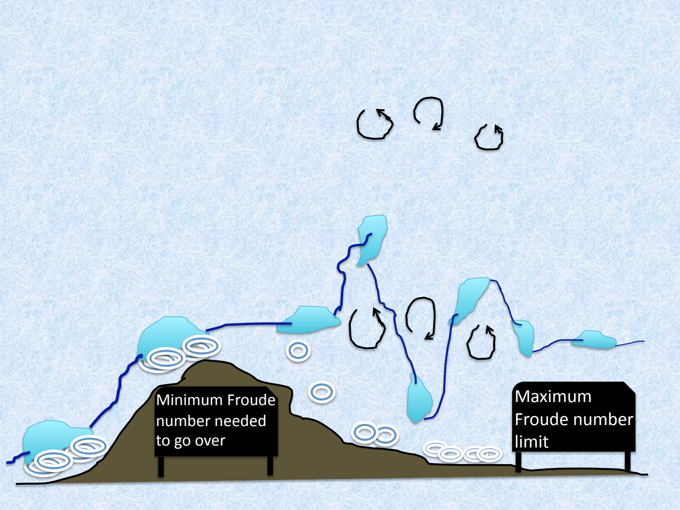
- How does wave drag impact the stratification, kinetic energy, and the input and output terms in the kinetic energy equation?
- Are general circulation ocean models forced only by winds and air-sea fluxes improved when wave drag is included?

What is topographic wave drag? (Froude number $= U/NH$)



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Parallel Ocean Program (POP) component of the Community Earth System Model (CESM) 1.1

- 62 z-layers
- $1/10^o$ resolution

Energy Inputs and Outputs

Inputs

- **Air-sea fluxes** - monthly mean ECMWF Re-Analysis (ERA-40; *Kallberg et al.*, 2004) for HYCOM, Coordinate Ocean Reference Experiment (CORE 2.0; *Large and Yeager*, 2009) for POP

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Dissipators

- **Horizontal viscosity** - ($\sim 10^2 - 10^3$ m^2 s^{-1}) includes the maximum of a Laplacian and a *Smagorinsky* (1993) parameterization with an additional biharmonic term for HYCOM, biharmonic term for POP

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- **Wave drag** - *Garner* (2005) scheme is used (see later)

Diagnostics informed by observations and compared with model output

Current meters (Global Multi-Archive Current Meter Database; http://stockage.univ-brest.fr/˜scott/GMACMD/updates.html)

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Satellite altimetry (Archiving, Validation and Interpretation of Satellite Oceanographic; http://www.aviso.oceanobs.com/es/data/index.html)

- Surface kinetic energy
- Eddy length scales (inverse first centroid of kinetic energy power spectrum)
- Sea surface height variance
- Intensified jet positions (via *Kelly et al.*, 2007)

Outline

1) What is the range of wavenumbers over which the internal waves are not evanescant?

$$f/U \sim 10^{-4}m^{-1} < |\vec{k}| < N/U \sim 10^{-1}m^{-1}. \quad (1)$$

Here,

- f is the Coriolis parameter
- N is the buoyancy frequency
- U is the velocity near the seafloor
- $|\vec{k}|$ is the wavenumber of the internal wave

Scott et al. (2011) used a range that went down to $f/U \sim 10^{-6}$ m^{-1}.

2) Which wave drag parameterizations are there to choose from?

Using a momentum sink:

- Implement in wavenumber space; e.g., *Bell* (1975)
- Implement in physical space; e.g., *Garner* (2005)

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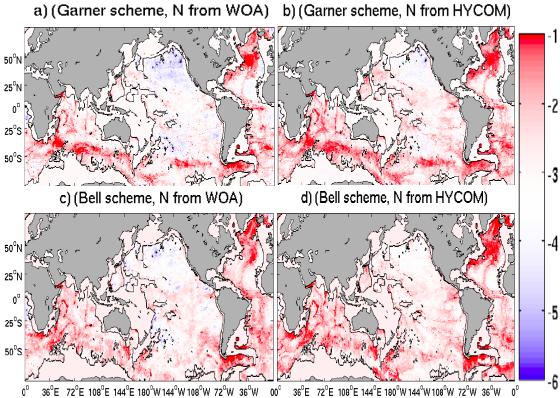
Features of *Garner* (2005) vs those of *Bell* (1975)

- *Garner* (2005) - allows for topographic blocking, but does not depend on Coriolis
- *Bell* (1975) - does not allow for topographic blocking, but does depend on Coriolis
- Both schemes - depend on stratification, velocity, and underlying topographic features and assume $f \ll N$

2) (cont...) Comparison of the *Bell* (1975) and *Garner* (2005) schemes

We choose to use the *Garner* (2005) scheme, but the *Bell* (1975) scheme yields similar results (offline)



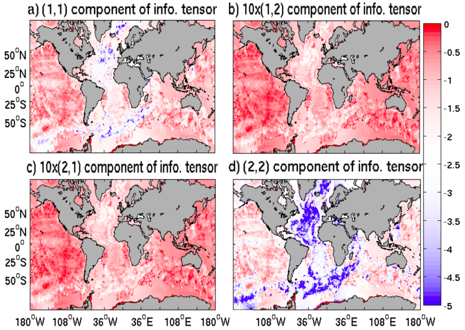
3) Where do we apply wave drag?

- Is the model numerically stable when wave drag is applied everywhere?
- Is it possible and does it make sense to apply wave drag everywhere?

Interpolate over topographic slopes that are supercritical? Apply wave drag only in abyssal hill regions? Apply wave drag only in regions deeper than 500 meters? . . .

4) Estimate the input parameters for the wave drag scheme of your choice

- Integrate *Goff and Jordan* (1988) abyssal hill power spectrum, weighted by wavenumbers from **(1)**
- parameters for power spectrum from *Goff and Arbic* (2010) and *Goff* (2010) in abyssal hill regions
- use a machine learning algorithm (*Wood*, 2006) to fill in the non-abyssal hill regions



5) How should the momentum be deposited vertically?

- Is there observational evidence for enhanced turbulence, if not lee wave drag, in the bottom, say, 500 meters? (see *Naveira-Garabato et al.*, 2012)
- Is there evidence that there needs to be a depth-dependent vertical deposition of momentum? (*Polzin* (2009) suggests that there is and the *Garner* (2005) scheme is capable of doing this)
- Are there locations where a non-trivial vertical deposition of momentum is important? (will not be addressed here)

Outline

Momentum equations → kinetic energy equation

$$
\begin{aligned}
&\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u} + \frac{1}{\rho}\vec{\nabla}p + f\hat{k}\times\vec{u} + g\hat{k} = \\
&\frac{\delta_s}{\rho}\frac{\vec{\tau}_{wind}}{H_s} - \delta_{b,H_{BD}}\frac{C_d}{H_{BD}}|\vec{u}|\vec{u} - \delta_{b,H_{WD}}\frac{|r_{drag}|}{H_{WD}}\vec{u} \\
&-\frac{\partial}{\partial z}(\nu_z\frac{\partial}{\partial z}\vec{u}_H) - \vec{\nabla}\cdot(\nu_{h,2}\vec{\nabla}\vec{u}_H + \nu_{h,4}\vec{\nabla}\nabla^2\vec{u}_H)
\end{aligned}
\tag{2}
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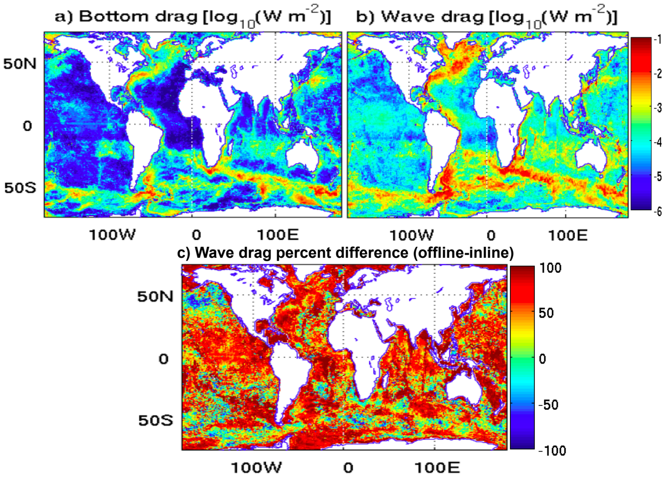
$$
P_{E_K time} + P_{E_K advection} = P_{pressure} + P_{input} - P_{output} + C_{E_K \to E_P} \tag{3}
$$

Bottom and wave drag



Global Integrals of Input/Output Terms in TW$= 10^{12}$W

$$P_{E_K time} + P_{E_K advection} = C_{E_K \to E_P} + P_{pressure} + P_{Wind} - P_{BD} - P_{WD} - P_{VV} - P_{HV} \quad (4)$$

WD?	Wind	Buoy	BD	WD	VV	HV
no	0.87	0.066	0.31	N/A	0.29	0.29
yes	0.87	0.066	0.14	0.40	0.28	0.26

Inputs vs Outputs:

- 5% imbalance (outputs less than inputs) without wave drag
- 15% imbalance (inputs less than outputs) with wave drag

Mass conservation equation → potential energy equation

$$\int dV \frac{d(\rho g z)}{dt} = \int dV \left[\frac{\partial(\rho g z)}{\partial t} + \vec{u} \cdot \vec{\nabla}(\rho g z) \right] \tag{5}$$

Mass conservation equation → potential energy equation

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$$\int dV \frac{d(\rho g z)}{dt} = \int dV \left[\rho \frac{d(gz)}{dt} + \frac{d\rho}{dt}(gz) \right] \tag{6}$$
$$= \int dV \, [\rho g w] + \int dx \int dy \left[g \eta \kappa \frac{\partial \rho}{\partial z} \right] - \int dV \left[g \kappa \frac{\partial \rho}{\partial z} \right]$$

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$$P_{E_P time} + P_{E_P advection} = P_{diffusive} + C_{E_P \rightarrow E_K} + C_{E_I \rightarrow E_P} \qquad (7)$$

Global Integrals of Mechanical Energy Budget Terms in TW$= 10^{12}$W

$$P_{E_K time} + P_{E_P time} + P_{E_K advection} + P_{E_P advection} = P_{pressure} + P_{diffusive} + P_{input} - P_{output} + C_{E_I \to E_P} \tag{8}$$

$KEadv.$	$PEadv.$	$press.$	$diffuse$	$E_I \to E_P$	$input$	$output$
-.00284	.174	$<$ .001	.00309	.0865	.868	1.06

7% imbalance of mechanical energy budget we ignore:

- partial time derivatives of KE and PE
- along-isopycnal contributions to power associated with buoyancy diffusion
- compressibility

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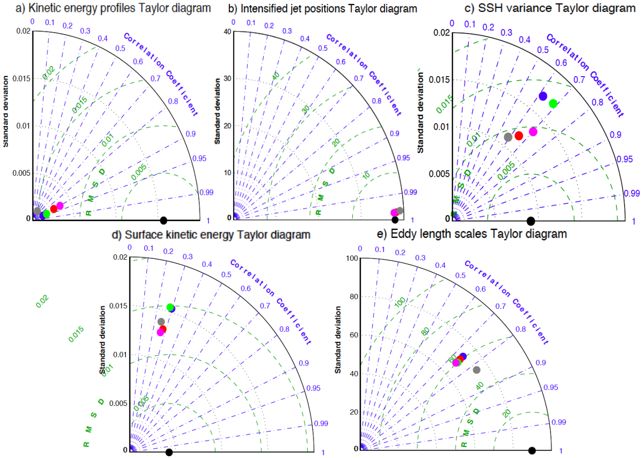
Taylor diagrams of all five diagnostics

Does wave drag ever make the model simulations in worse agreement with diagnostics informed by observations?

Observations, $1/12^o$ HYCOM without wave drag, $1/12^o$ HYCOM with wave drag, $1/25^o$ HYCOM without wave drag, $1/25^o$ HYCOM with wave drag, $1/10^o$ POP without wave drag (*Taylor*, 2001)



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- what's the more appropriate wave drag scheme to use and in what context?
- use of the full wave drag tensor that *Garner* (2005) formulated?
- use of a depth-dependent momentum deposition procedure that *Garner* (2005) formulated?
- use of an alternative, non-local momentum deposition procedure?

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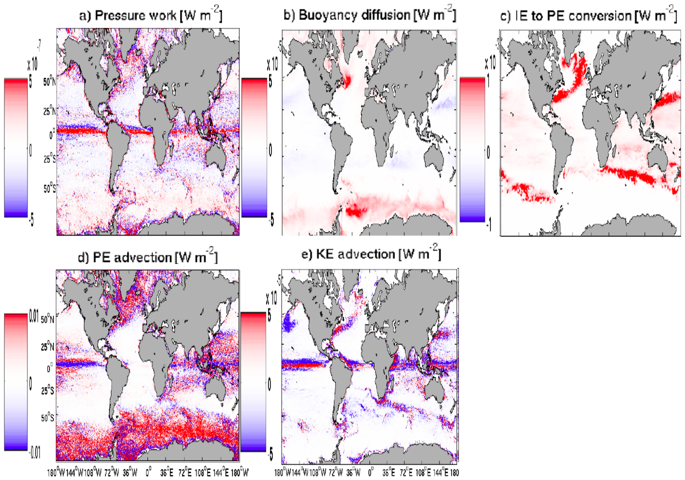
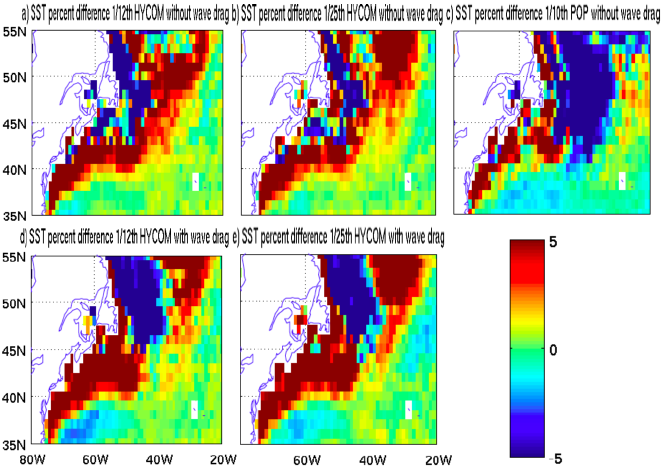
- substantially less bottom drag dissipation with wave drag, and wave drag cannot be substituted for by boosting bottom drag
- all other mechanical energy budget terms are spatially altered, but changed by little in their global integrals
- wave drag either improves the model or does not make the model worse

Non-input/output mechanical energy budget terms



SST bias


a) SST percent difference 1/12th HYCOM without wave drag
b) SST percent difference 1/25th HYCOM without wave drag
c) SST percent difference 1/10th POP without wave drag
d) SST percent difference 1/12th HYCOM with wave drag
e) SST percent difference 1/25th HYCOM with wave drag
55N
50N
45N
40N
35N
80W
60W
40W
20W
5
0
-5

SSH variance