GENERALIZED COORDINATE TREATMENT IN HYCOM

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General remarks

An experimental scheme is described that allows coordinate layers in MICOM to become nonisopycnic wherever shallow depth or lack of stratification causes isopycnic layers to collapse to zero thickness at the bottom of the mixed layer. The resulting "hybrid" coordinate system puts previously deactivated isopycnic grid points to use in improving vertical model resolution.

The hybrid coordinate approach has been in operational use for several years in short-range numerical weather prediction ("Rapid Update Cycle", http://maps.fsl.noaa.gov/), but not much has been known until recently about the behavior of hybrid models in long-term integrations.

The biggest challenge to an oceanic hybrid coordinate scheme is the annual buildup and obliteration of the seasonal thermocline. Warming and cooling of the ocean from above creates hybrid coordinate layers that are isopycnal in character during summer but change into geometrically defined layers in winter. Considerable freedom exists in placing the latter in the water column. Allowing geometrically defined layers to stay near the surface during winter minimizes vertical grid migration and concomitant dispersion of water mass properties; allowing them to descend to greater depths in winter improves vertical grid spacing in deep mixed layers. These tradeoffs are currrently being investigated.

Specifics

Consider three consecutive isopycnic layers, labeled 0, 1, 2, in a stratified water column. Let $\alpha_k > \alpha_{k+1}$ where α is the specific volume. Label the interfaces such that interface k denotes the *upper* interface of layer k.

Suppose α_1 differs from its layer reference value $\widehat{\alpha}_1$. We seek ways to re-discretize the water column in a manner which preserves the overall height of the column, represented by the integral $\int \alpha dp$, while changing α_1 to $\widehat{\alpha}_1$. Conservation of the integral requires that alterations of α_1 must be accompanied by vertical displacement (in p space) of one or more interfaces. The simplest solution is to move only one interface and to select this interface in the spirit of the "donor cell" transport scheme. By this we mean that the upper interface is moved upward, i.e., layer 1 is diluted with lighter water from layer 0, if layer 1 is too dense ($\alpha_1 < \widehat{\alpha}_1$). Analogously, the lower interface is moved downward, i.e., layer 1 is diluted with denser water from layer 2, if layer 1 is too light ($\alpha_1 > \widehat{\alpha}_1$). This method of restoration does not work in the special case where the coordinate layer touching the sea floor is too light, and like all upstream advection schemes it is inherently diffusive. Methods for conserving the integral $\int \alpha dp$ by extruding water to a neighboring layer, rather than entraining water, will be discussed later.

<u>*Case 1:* $\alpha_1 < \widehat{\alpha}_1$</u>

This is the case where the *upper* interface is moved, i.e., mass is exchanged between layers

0 and 1. Conservation of $\int \alpha dp$ requires

$$\alpha_0(p_1 - p_0) + \alpha_1(p_2 - p_1) = \alpha_0(\widehat{p}_1 - p_0) + \widehat{\alpha}_1(p_2 - \widehat{p}_1) \tag{1}$$

where $\widehat{p}_1$ is the pressure of the upper interface after re-discretization. Solving (1) for $\widehat{p}_1$ yields the expression

$$\widehat{p}_1 = \frac{p_1(\alpha_0 - \alpha_1) + p_2(\alpha_1 - \widehat{\alpha}_1)}{\alpha_0 - \widehat{\alpha}_1}. \tag{2}$$

Note that the weight assigned to p_2 is negative, indicating that for large discrepancies between α_1 and $\widehat{\alpha}_1$ (2) will not necessarily yield a solution $\widehat{p}_1 > p_0$. To assure a minimum interface spacing, we therefore replace $\widehat{p}_1$ by

$$\tilde{p}_1 = \max(\widehat{p}_1, p_0 + \Delta_0) \tag{3}$$

where Δ_0 is a minimum layer thickness. Moving the interface to $\tilde{p}_1$ instead of $\widehat{p}_1$ means, of course, that layer 1 will end up with a specific volume $\tilde{\alpha}_1$ different from the reference value $\widehat{\alpha}_1$.

Following Bleck and Boudra (1981), Δ_0 is chosen to be a continuously differentiable function which for large positive arguments $\Delta p \equiv \widehat{p}_1 - p_0$ returns the argument Δp (meaning that $\tilde{p}_1 = \widehat{p}_1$), while for large negative arguments it returns a small constant δ representing the lowest permissible layer thickness. Our traditional choice for this so-called "cushion" function (Bleck and Benjamin, 1993) is

$$\Delta_0 = \begin{cases} \Delta p & \text{if} \quad \frac{\Delta p}{\delta} > 4 \\ \delta\left[1 + \frac{1}{3}\left(\frac{\Delta p}{2\delta} + 1\right)^2\right] & \text{if} \quad -2 < \frac{\Delta p}{\delta} < 4 \\ \delta & \text{if} \quad -2 > \frac{\Delta p}{\delta} \end{cases} \tag{4}$$

which is obtained by fitting a piece of a parabola in $(\Delta p, \Delta_0)$ space between the two straight lines $\Delta_0 = \Delta p$ and $\Delta_0 = \delta$. Another example from this class of spliced-together cushion functions, this one acting over a somewhat wider range of $\Delta p/\delta$, is

$$\Delta_0 = \begin{cases} \Delta p & \text{if} \quad \frac{\Delta p}{\delta} > 6 \\ \delta\left[1 + \frac{4}{5}\left(\frac{\Delta p}{4\delta} + 1\right)^2\right] & \text{if} \quad -4 < \frac{\Delta p}{\delta} < 6 \\ \delta & \text{if} \quad -4 > \frac{\Delta p}{\delta} \end{cases} \tag{5}$$

Note that the above cushion functions, whose purpose it is to smooth the transition between isopycnally and geometrically constrained segments of a coordinate surface, make no reference to neighboring grid points. This simplicity lends a degree of robustness to our hybrid scheme not necessarily found in more complicated formulas.

Two cases must be distinguished in computing $\tilde{\alpha}_1$ from (1):

Case 1a: $\tilde{p}_1 < p_1$ In this case, the interface ends up at a lower pressure, allowing lighter water to raise α_1 to a value $\tilde{\alpha}_1$ closer – if not equal – to the reference value $\widehat{\alpha}_1$. The value $\tilde{\alpha}_1$ is obtained by substituting tildes for carats in (1):

$$\tilde{\alpha}_1 = \frac{\alpha_1(p_2 - p_1) + \alpha_0(p_1 - \tilde{p}_1)}{p_2 - \tilde{p}_1}. \tag{6}$$

Case 1b: $\tilde{p}_1 > p_1$ In this case, layer 1 "donates" water to layer 0; hence, it is α_0, rather than α_1, that is being modified. Conservation of $\int \alpha dp$ then leads to

$$\tilde{\alpha}_0 = \frac{\alpha_1(\tilde{p}_1 - p_0) + \alpha_0(p_1 - p_0)}{\tilde{p}_1 - p_0}. \tag{7}$$

Before solving (7), $\tilde{p}_1$ must be modified, if necessary, to satisfy $\tilde{p}_1 \le p_2 - \Delta_2$ where Δ_2 is a layer thickness threshold presently set to $(p_2 - p_1)/2$.

Case 2: $\alpha_1 > \widehat{\alpha}_1$

This is the situation where we decide to move the *lower* interface. The equation replacing (1) in this case is

$$\alpha_1(p_2 - p_1) + \alpha_2(p_3 - p_2) = \widehat{\alpha}_1(\widehat{p}_2 - p_1) + \alpha_2(p_3 - \widehat{p}_2). \tag{8}$$

It calls for moving the pressure of the lower interface from p_2 to

$$\widehat{p}_2 = \frac{p_1(\widehat{\alpha}_1 - \alpha_1) + p_2(\alpha_1 - \alpha_2)}{\widehat{\alpha}_1 - \alpha_2}. \tag{9}$$

The concern in this case is that $\widehat{p}_2$ may get too close to, or even exceed, p_3. We therefore replace $\widehat{p}_2$ by

$$\tilde{p}_2 = \min(\widehat{p}_2, p_3 - \Delta_3), \tag{10}$$

with Δ_3 currently set to $(p_3 - p_2)/2$. Replacing $\widehat{p}_2$ by $\tilde{p}_2$ in (8) changes $\widehat{\alpha}_1$ into

$$\tilde{\alpha}_1 = \frac{\alpha_2(\tilde{p}_2 - p_2) + \alpha_1(p_2 - p_1)}{\tilde{p}_2 - p_1}. \tag{11}$$

Case 2a: $\alpha_1 > \widehat{\alpha}_1$ where layer 1 is the bottom layer

Since the lower interface cannot move down in this case, we use (1) and compute an upper interface movement from (2). While it may sound physically implausible that the density in a layer can be manipulated by extruding water into an adjacent layer, this operation is legitimate if one conceptually allows density to vary vertically within a coordinate layer.

Specifically, as long as $\alpha_0 > \alpha_1 (> \widehat{\alpha_1})$, layer 1 may be viewed as being composed of two sublayers, one of specific volume α_0 and one of specific volume $\widehat{\alpha_1}$. Having the same density as layer 0, the lighter sublayer can be merged with layer 0. The denser sublayer, which has the desired density $\widehat{\alpha_1}$, stays in layer 1.

To keep $\widehat{p}_1$ from exceeding the bottom pressure p_2, we replace $\widehat{p}_1$ by

$$\tilde{p}_1 = \min(\widehat{p}_1, p_2 - \Delta_2) \tag{12}$$

with Δ_2 currently set to $(p_2 - p_1)/2$.

An important question is how to redistribute T and S among the layers. One possible approach is to translate $\widehat{p}_k - p_k$ into a generalized vertical velocity (i.e., vertical motion relative to the layer interface) and to advect T and S accordingly. Ideally, this vertical advection process should (a) conserve T, S in the column, and (b) produce T, S changes in layer 1 that satisfy (1).

A scheme that exactly satisfies (a) while approximately satisfying (b) can be obtained by reapportioning T, S among the layers in a manner analogous to (6),(7),(11), whichever is applicable. In *Case 1a*, for example, where layer 1 properties are being modified by adding water from layer 0, this means

$$\tilde{T}_1 = \frac{T_1(p_2 - p_1) + T_0(p_1 - \tilde{p}_1)}{p_2 - \tilde{p}_1}$$
$$\tilde{S}_1 = \frac{S_1(p_2 - p_1) + S_0(p_1 - \tilde{p}_1)}{p_2 - \tilde{p}_1}.$$

The new values $\tilde{T}_1, \tilde{S}_1$ would combine to produce the desired specific volume $\tilde{\alpha}_1$ in layer 1 if the equation of state were linear, i.e., if

$$\tilde{\alpha}_1 - \alpha_1 = \left(\frac{\partial \alpha}{\partial T}\right)_S (\tilde{T}_1 - T_1) + \left(\frac{\partial \alpha}{\partial S}\right)_T (\tilde{S}_1 - S_1)$$

were exactly satisfied. This not being the case, the specific volume implied by $\tilde{T}_1, \tilde{S}_1$ will differ slightly from the prescribed value $\tilde{\alpha}_1$. In cases where $\tilde{\alpha}_1$ equals the layer reference value $\widehat{\alpha}_1$, this difference prevents the scheme from exactly restoring isopycnic conditions in one instant. This disadvantage is likely to be inconsequential, however, because repetitive use of the coordinate restoration scheme presented here will quickly reduce the residual difference to zero.

Division of a coordinate layer into two sublayers of differing densities and extruding one for the purpose of altering the layer's original density requires special consideration. The strategy presently adopted in Case 2a is to unmix T and S in a direction *perpendicular* to the isopycnal α_1 in T/S space. Details are as follows.

Define nondimensional temperature and salinity as $T' = T/T_{scl}$ and $S' = S/S_{scl}$. Setting the left-hand side in the expression

$$d\alpha = \frac{\partial \alpha}{\partial T'} dT' + \frac{\partial \alpha}{\partial S'} dS'$$

to zero reveals that the vector (dT', dS') pointing in the direction *tangential* to an isopycnal in T/S space must be perpendicular to the vector $(\partial\alpha/\partial T', \partial\alpha/\partial S')$. Hence, the direction *normal* to the isopycnal is perpendicular to the vector $(\partial\alpha/\partial S', -\partial\alpha/\partial T')$. This can be stated in the form

$$\frac{dS'}{\partial\alpha/\partial S'} = \frac{dT'}{\partial\alpha/\partial T'}.$$

Combining the last two expressions, we find that the increments (dT', dS') changing the specific volume α to $\alpha + d\alpha$ in a direction perpendicular to the isopycnal α are given by

$$dT' = d\alpha \, \frac{\partial\alpha/\partial T'}{(\partial\alpha/\partial T')^2 + (\partial\alpha/\partial S')^2} \tag{13}$$

$$dS' = d\alpha \, \frac{\partial\alpha/\partial S'}{(\partial\alpha/\partial T')^2 + (\partial\alpha/\partial S')^2}. \tag{14}$$

Defining T, S in the two sublayers in case 2a is now simply a matter of defining $d\alpha$ in (13),(14). To set the T/S properties in the lower sublayer, we set $d\alpha = \widehat{\alpha}_1 - \alpha_1$ while in the upper sublayer we set $d\alpha = \alpha_0 - \alpha_1$.

Note that "perpendicularity" in T/S space depends on the scaling values T_{scl}, S_{scl}. The relative contribution of T and S unmixing to the creation of the required density contrast between the sublayers can be controlled by these scaling values. For example, as $T_{scl}/S_{scl} \to \infty$, density contrast is achieved entirely by T unmixing.

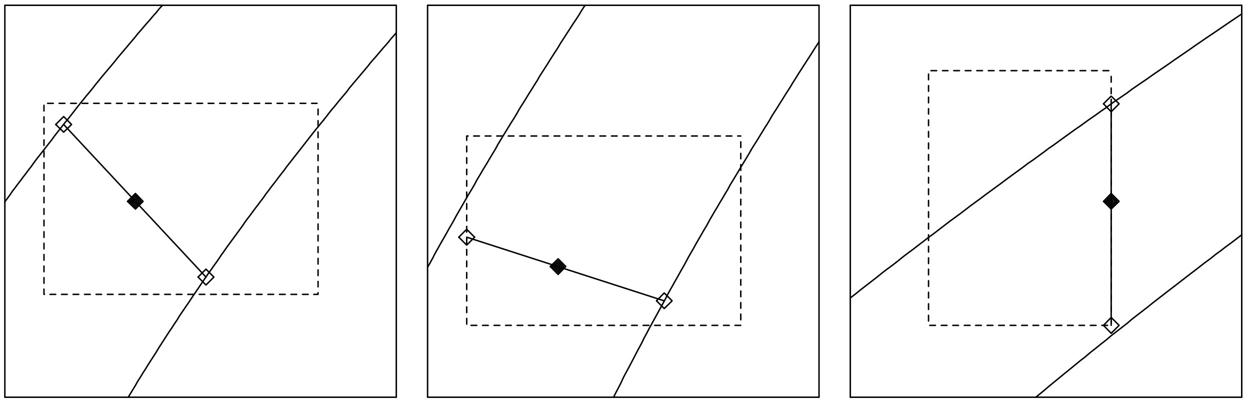
With the exception of Case 2a, the density restoration method outlined in this note may be viewed as an upstream or donor cell advection scheme whose diffusive character is a possible concern. Schemes that modify the density of both the receiving and the donating layer (an attribute found in the less diffusive centered-differencing schemes) offer a way to reduce this numerical diffusion. The density restoration scheme sketched below attempts to mimic the diffusion properties of centered differencing in a manner similar to the approach taken in Case 2a. Specifically, this scheme divides layer 1 into two sublayers, one of which has the desired target density $\widehat{\alpha}_1$. The second sublayer is then expelled into either the layer above or below, depending on whether the second sublayer is lighter or denser than the first one.

Of utmost importance in the design of such an "unmixing" scheme is that it will not to create artificial T, S extrema in the water column. To guarantee this, we require the unmixing process to be confined to a "bounding box" in T/S space whose four sides are defined in terms of T, S values in layers 0 and 2. Since the purpose of unmixing is to create sublayers of prescribed density, one or two corners the bounding box may in practice be truncated by isopycnals representing the target densities of the two sublayers.

The sublayer generation routine presently in use treats the target density and T, S constraints as independent and requires that both be satisfied. The scheme tests two "candidate" unmixing directions as to their ability to create sublayers of the desired densities. Both unmixing directions are defined as lines that connect the T, S point representing the original layer 1 in T/S space with either the upper left or lower right corner of the rectangular bounding box. In either case, the unmixing line terminates on both ends on either the target isopycnal or the edge of the bounding box, whichever is encountered first. The unmixing direction ultimatey chosen is the one that results in the bigger density contrast. Knowing the end points of the unmixing line, we can compute the thickness of the two sublayers by requiring T, S conservation during unmixing.

Examples of unmixing scenarios are shown in the attached figure. Abscissa and ordinate in the three plots are salinity and temperature, respectively. The bounding box is dashed while the slanting curves represent isopycnals. The black diamond represents the layer to be unmixed, while the hollow diamonds represent the two resulting sublayers.

Aside from reducing numerical diffusion, this unmixing scheme removes an undesirable bias from the coordinate restoration logic, namely, that layers subject to density restoration invariably *grow* at the expense of their neighbors. The present unmixing scheme has the opposite effect, i.e., it causes layers whose density is being manipulated to *shrink*. We use the cushion function described earlier to prevent a layer from getting too thin in the process. Our present strategy is to attempt partial coordinate restoration through unmixing before resorting to the donor cell method desribed earlier. The exact meaning of the word "partial" remains to be determined.



For further details of our present implementation of the hybrid grid generator, the reader is referred to the source listing of subroutine `hybgen.f`. Routine `hybgen.f` is intended to be called from within `hycom.f` following the calls to the "physics" routines `cnuity.f`, `tsadvc.f`, ..., `mxlayr.f` (several of which represent departures from MICOM routines to account for the nonisopycnic character of interior coordinate layers.)

In the present implementation of `hybgen.f`, the parameter δ appearing in (4) and (5) is set to 10 m. Except in very shallow water, it is advisable to make δ a constant. This will prevent the potential-vorticity conserving advection and Coriolis terms in the momentum equations, which are written in a shallow-water form inherited from MICOM, from generating spurious stretching effects. Keeping coordinate surfaces horizontal in geographic regions where layers are unable to retain their isopycnal character, i.e., where coordinate surfaces are not material under adiabatic flow conditions, assures that layer thickness drops out of the momentum advection and Coriolis terms, allowing them to act the way they would in Cartesian coordinate models.

In shallow coastal areas, on the other hand, it is appropriate to scale δ by the total water depth. The argument here is that Taylor columns in the coastal zone are likely to span the whole water depth, implying that undulations in the depth of σ surfaces (σ = depth scaled by bottom depth) accurately portray the stretching effect due to sea floor undulations.

References

Bleck, R., and D. B. Boudra, 1981: Initial testing of a numerical ocean circulation model using a hybrid (quasi-isopycnic) vertical coordinate. *J. Phys. Oceanogr., 11*, 755–770.

Bleck, R., and S.G. Benjamin, 1993: Regional weather prediction with a model combining terrain-following and isentropic coordinates. Part 1: model description. *Mon. Wea. Rev., 121*, 1770–1785.