Available Potential Energy in the ocean – what is it good for?

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Climate sensitivity: SAT change due to CO₂ doubling

Rate of Deep Ocean Heat-uptake [Sqrt(K,v)]

(K_v based on ratio of SSH change to SAT change)

Courtesy: Peter Stone, MIT
Applications

• Is APE a useful indicator of model drift?
• Which processes contribute to the generation and destruction of APE?
• Do processes like vertical regridding in HYCOM distort the APE balance?

(Turn page for derivation of APE….)
Potential energy per unit column of a fluid column in hydrostatic balance:

\[ PE = \int_{z_{bot}}^{z_{top}} g \rho z \, dz = \int_{p_{bot}}^{p_{top}} z \, dp \]

where \( \text{bot}, \text{top} \) denote bottom and top of fluid column. Integration by parts yields

\[ PE = (zp)_{bot} + \int_{z_{bot}}^{z_{top}} p \, dz = (zp)_{bot} + \frac{1}{g} \int_{p_{bot}}^{p_{top}} \alpha \, d \frac{p^2}{2} \]

where \( \alpha = \rho^{-1} \)
Conversion to \( \mathbf{s} \) coordinates, global integration:

\[
PE = (zp)_{bot} + \frac{1}{g}_{\text{stop}} \int sbot \frac{\partial}{\partial s} \frac{p^2}{2} \, ds
\]

where

\[
q = \frac{\int_{\text{globe}} q \, dx \, dy}{\int_{\text{globe}} dx \, dy}
\]

Define mass-weighted average:

\[
\tilde{q} = q \frac{\partial p}{\partial s} / \frac{\partial p}{\partial s}
\]
With \( q = \bar{q} + q^* \sim q + q' \)

the integrand can be split 3 ways:

\[
\alpha \frac{\partial}{\partial s} \frac{p^2}{2} = \sim \alpha \frac{\partial}{\partial s} \frac{\bar{p}^2}{2} + \sim \alpha \frac{\partial}{\partial s} \frac{p^{*2}}{2} + \alpha' \frac{\partial}{\partial s} \frac{p^2}{2}
\]

If evaluated in \textit{density} space (\( \alpha' = 0 \)), the r.h.s. reduces to

\[
\alpha \frac{\partial}{\partial s} \frac{\bar{p}^2}{2} + \alpha \frac{\partial}{\partial s} \frac{p^{*2}}{2}
\]

Unavailable 

pot. energy

Available 

pot. energy
A 3-layer ocean. $p$ shown in red
Procedure for determining $\bar{p}$:

1. Compute the mass $M$ above a given $s$ surface.
2. Divide ocean into large number of thin isobaric slabs; determine mass $m_k$ in each slab from bathymetric data base.
3. Counting from the top down, find the index $n$ satisfying

$$\sum_{k=1}^{n-1} m_k \leq M \leq \sum_{k=1}^{n} m_k$$

4. $\bar{p}$ is now known to lie between top and bottom of slab $n$. Interpolate to get final value.