Embedding a forward model of barotropic and baroclinic tides into HYCOM

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Will be working with Eric, Harley, Joe, and Alan to insert tides into HYCOM.

Builds on:

Arbic, Garner, Hallberg, Simmons (2004)

Simmons, Hallberg, Arbic (2004)

Arbic, MacAyeal, Mitrovica, Milne (2004)

Arbic (2005)

Motivation

Models of tides and non-tidal motions currently run separately—why not simultaneously?

Both tides and wind-driven motions provide energy sources for mixing, which affects large scale circulation.

Would like to have model including both, which derives mixing from dissipation, and dissipation from drag acting on both types of motions.

See poster for work on dissipation of eddies in idealized models.

Here describe work done on forward tide modeling, which has recently made great strides.

Tidal forcing periodic in time, has simple structure in space (Legendre polynomials), thus is good test of general circulation models.

Tidal dissipation and forward models

Total dissipation inferred from astronomy 3.7 TW $\Rightarrow$ 10 mW m^{-2} areal average.

Models with only quadratic drag put all dissipation into shallow seas:

$< \rho_0 c_d |\vec{u}|^3 > =$
0.02 mW m^{-2}, $|\vec{u}| = 2$ cm s^{-1},
323 mW m^{-2}, $|\vec{u}| = 50$ cm s^{-1}.

Alternative view: internal wave breaking over rough topography significant energy sink.

Egbert and Ray (2000, 2001): T/P-constrained models yield $\sim$ 1 TW dissipation over mid-ocean rough topography, in agreement with in-situ evidence (e.g. Polzin et al. 1997).

Jayne and St. Laurent (2001), Carrere and Lyard (2003), Egbert et al (2004), Arbic et al. (2004): accuracy of forward tidal models improved when topographic drag scheme is included.

Topographic drag schemes

JS: analytical result for wave drag for flow $\vec{u}$ over monochromatic terrain $hsin(kx)$, buoyancy frequency N:

drag$=\frac{1}{2}Nkh^2\vec{u}$, h=rms residual of unresolved topography, k = tunable parameter.

We use scheme of Garner (2005), which builds on analytical result for drag on steady flow over arbitrary topography and includes scalings for nonlinear effects at bottom.

One-layer equations

$$\frac{\partial \eta}{\partial t} + \nabla \cdot [(H+\eta)\vec{u}] = 0$$

$$\frac{\partial \vec{u}}{\partial t} + (f+\zeta)\hat{k} \times \vec{u} = -g\nabla(\eta - \eta_{EQ} - \eta_{SAL})$$

$$-\nabla(\frac{1}{2}\vec{u}\cdot\vec{u}) + \frac{\nabla \cdot [K_H(H+\eta)\nabla\vec{u}]}{H+\eta} - \frac{c_d|\vec{u}|\vec{u}}{H+\eta} + \frac{\overline{T}\vec{u}}{\rho_0(H+\eta)}$$

H: resting thickness

η: perturbation surface elevation

$\vec{u}$: velocity

f: Coriolis parameter

$\zeta = \hat{k} \cdot (\nabla \times \vec{u})$

K_H: horizontal friction

c_d: quadratic drag coefficient

$\overline{T}$: topographic drag tensor

Periods and frequencies ($\omega = 2\pi/period$) of celestial motions and of tides

Mean solar day: 1 mean solar day, ω_0
Mean lunar day: 1.0351 mean solar days, ω_1
Sidereal month: 27.3217 mean solar days, ω_2
Tropical year: 365.2422 mean solar days, ω_3
Moon's perigee: 8.85 Julian years, ω_4 Sidereal day: 0.9973 solar days, $\omega_s = \omega_0 + \omega_3 = \omega_1 + \omega_2$

Frequencies of four largest semidurnal tides:

M_2: $2\omega_1$
S_2: $2\omega_0$
N_2: $2\omega_1 - \omega_2 + \omega_4$
K_2: $2\omega_s$

Frequencies of four largest diurnal tides:

K_1: ω_s
O_1: $\omega_1 - \omega_2$
P_1: $\omega_0 - \omega_3$
Q_1: ω_1-2$\omega_2 + \omega_4$

Frequencies of two largest long-period tides:

M_f: $2\omega_2$
M_m: $\omega_2 - \omega_4$

Tidal species

- Semidiurnal tides (M_2,S_2,N_2,K_2):

$$\eta_{EQ} = A(1 + k_2 - h_2)cos^2(\phi)cos(\omega t + 2\lambda),$$

- Diurnal tides (K_1,O_1,P_1,Q_1):

$$\eta_{EQ} = A(1 + k_2 - h_2)sin(2\phi)cos(\omega t + \lambda),$$

- Long-period tides (M_f,M_m):

$$\eta_{EQ} = A(1 + k_2 - h_2)[\frac{1}{2} - \frac{3}{2}sin^2(\phi)]cos(\omega t),$$

where λ is longitude wrt Greenwich, ϕ is latitude, t is time wrt Greenwich, and A and ω are consituent-dependent amplitudes and frequencies.

- h_2: accounts for solid-earth body tide deformation
- k_2: accounts for change in potential due to self-attraction of solid-earth deformation
- $(1+k_2-h_2) = 0.693$ for semidiurnal and long period tides, $=$ up to 0.736 for diurnal tides due to "free-core nutation resonance" (Wahr 1981)

Self-attraction and loading

Earth yields to ocean loading. Potential altered by self-attractions of mass redistributions in earth and ocean (Hendershott 1972):

$$\eta_{SAL} = \sum_n \frac{3\rho_{water}}{\rho_{earth}(2n+1)}(1 + k'_n - h'_n)\eta_n$$

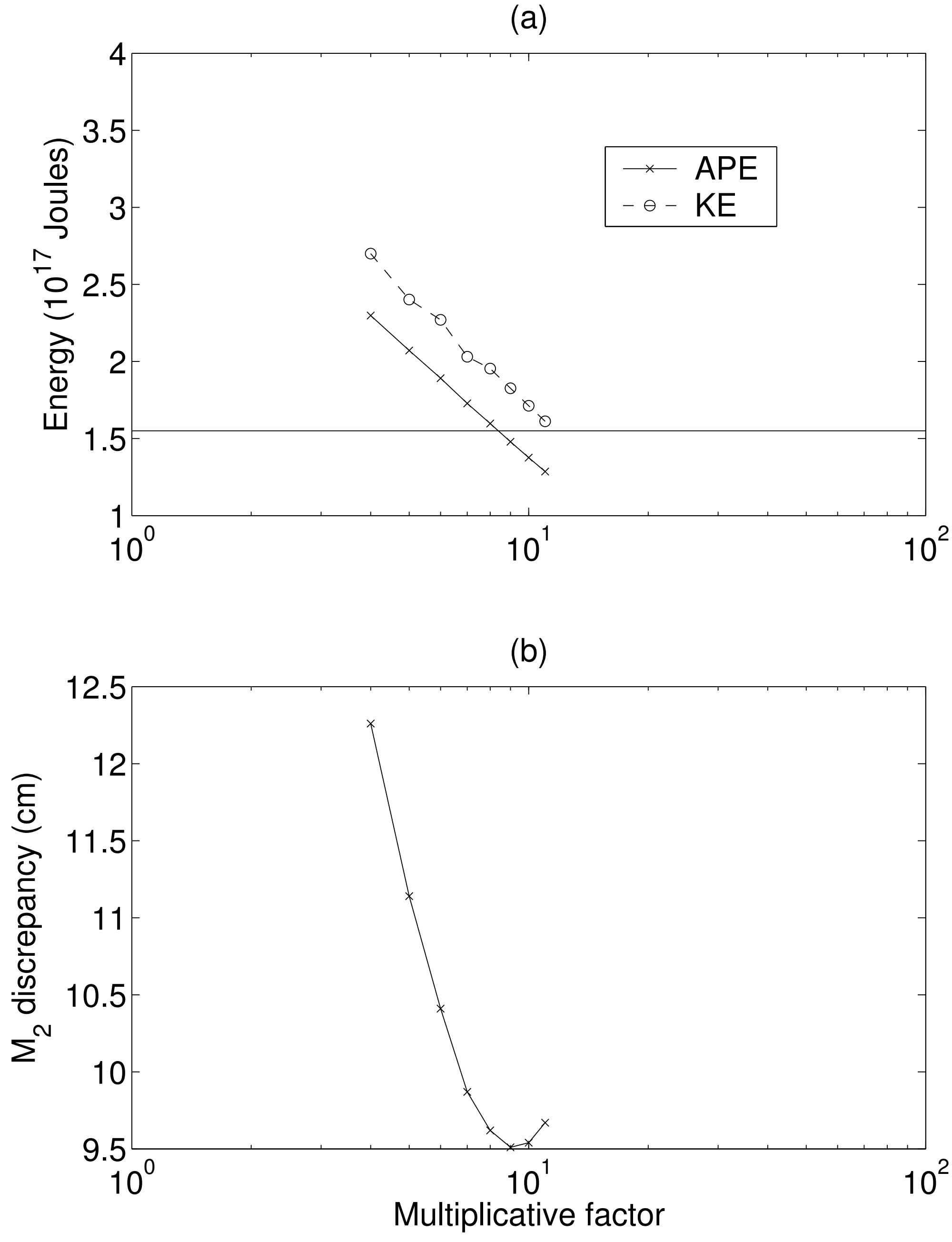
η_n nth spherical harmonic of η

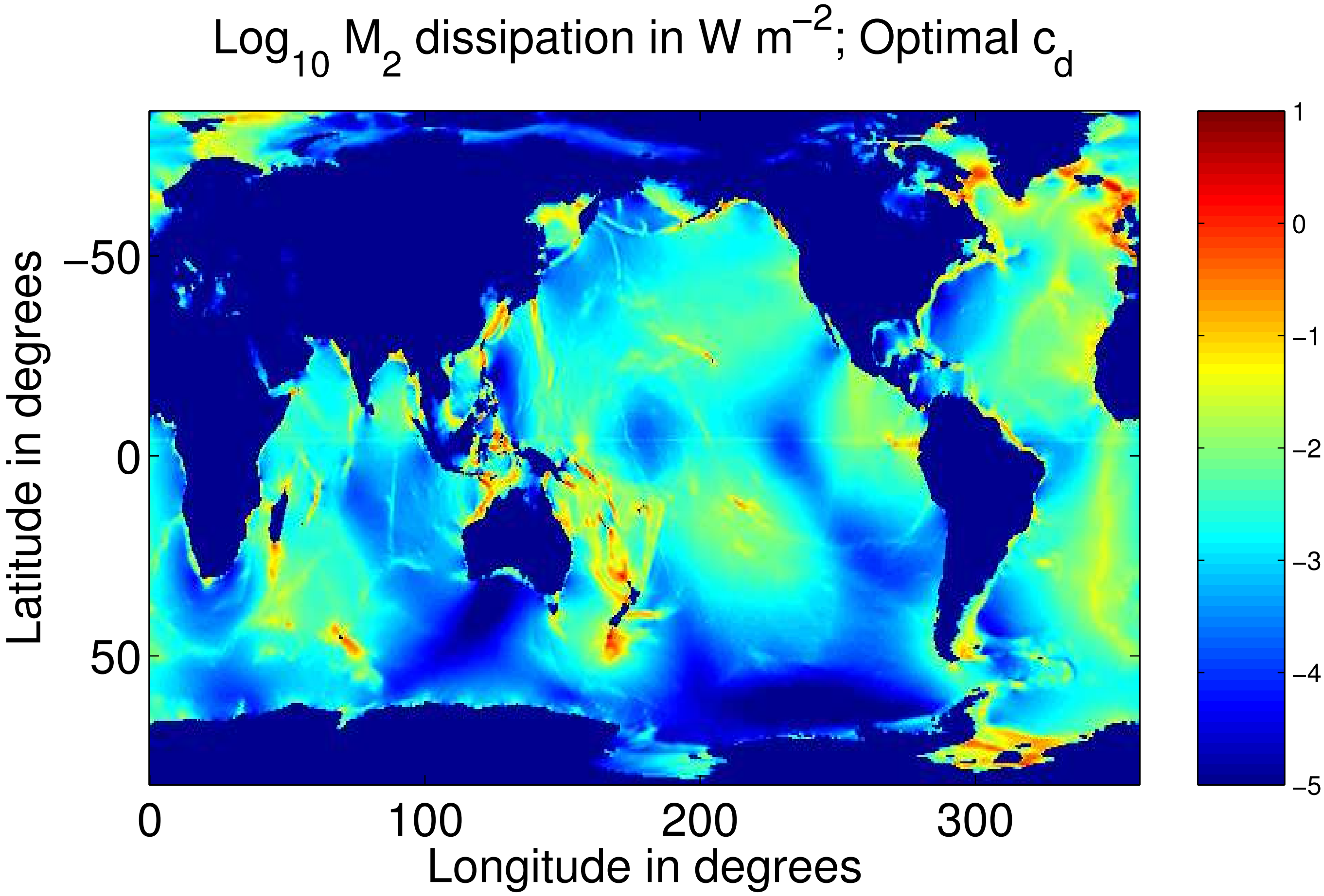
k'_n, h'_n load numbers (Munk and MacDonald 1960) from Farrell (1972)

Solve by iteration, starting with "scalar approximation" $\eta_{SAL} \approx 0.094\eta$.

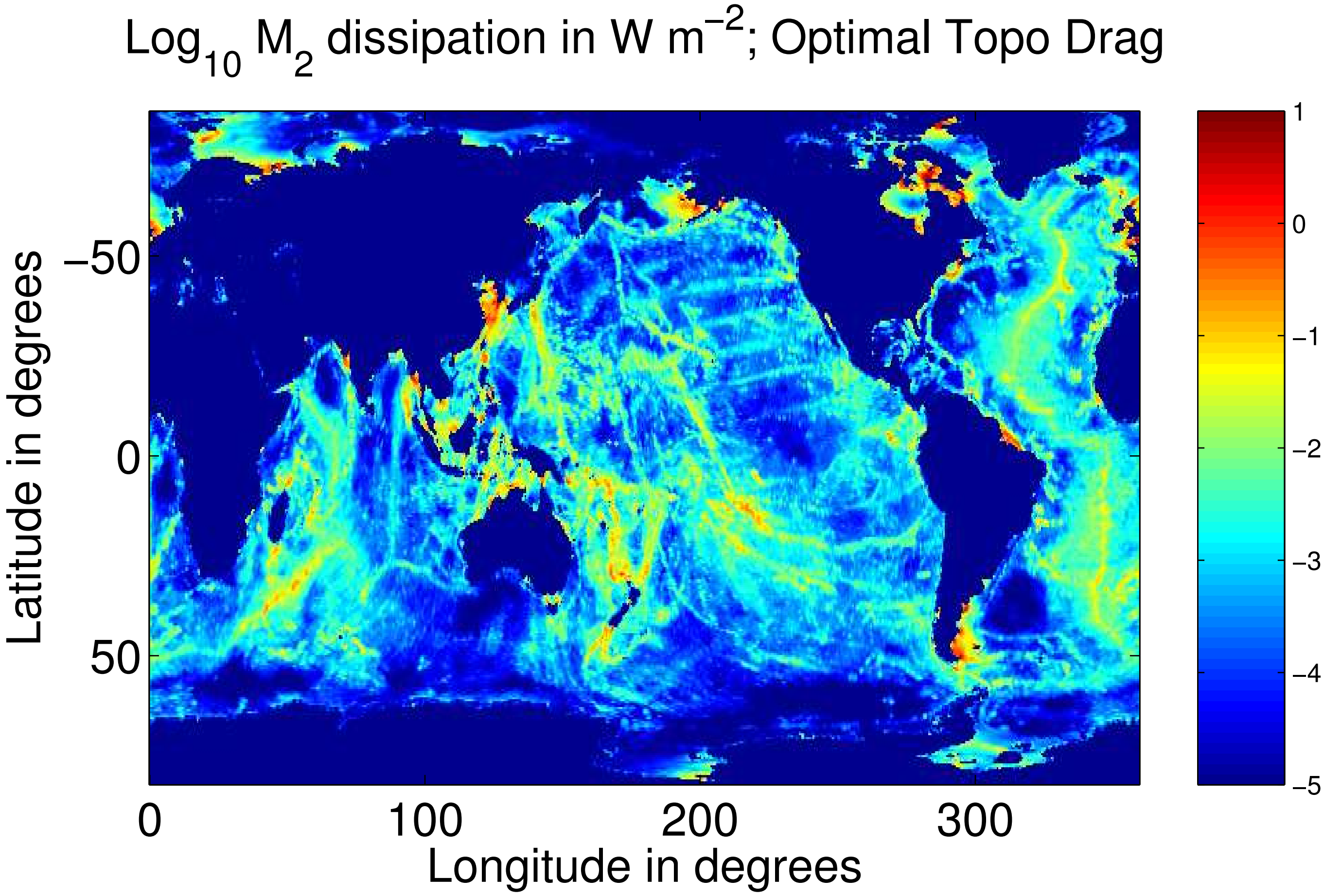
Attaining convergence in iteration non-trivial, use same trick that Egbert et al. (2004) used.

RMS M_2 elevation discrepancy between forward model with topographic wave drag and satellite-constrained model





Log_{10} M_2 dissipation in W m^{-2}; Optimal c_d
Latitude in degrees
−50
0
50
0
100
200
300
Longitude in degrees
1
0
−1
−2
−3
−4
−5



Log_{10} M_2 dissipation in W m^{-2}; Optimal Topo Drag
Latitude in degrees
−50
0
50
0
100
200
300
Longitude in degrees
1
0
−1
−2
−3
−4
−5

Elevation discrepancies (cm) of forward model, wrt to pelagic tide gauges

Without S_2 air tide forcing:

Const	Signal	One-layer model	Two-layer model
Q_1	1.62	0.43 (92.8)	0.40 (93.9)
O_1	7.76	1.79 (94.7)	1.62 (95.7)
P_1	3.62	0.84 (94.6)	0.73 (95.9)
K_1	11.26	2.90 (93.3)	2.26 (96.0)
N_2	6.86	1.95 (91.9)	1.83 (92.9)
M_2	33.22	9.33 (92.1)	8.75 (93.1)
S_2	12.66	4.40 (87.9)	4.21 (88.9)
K_2	3.43	1.00 (91.4)	0.96 (92.2)
RSS	39.06	11.12 (91.9)	10.34 (93.0)

With S_2 air tide forcing:

Const	Signal	One-layer model	Two-layer model
S_2	12.66	3.29 (93.2)	3.10 (94.0)
RSS	39.06	10.73 (92.5)	9.94 (93.5)

Things to do in HYCOM

Run and test HYCOM as tide-only model.

Run tides and non-tidal motions simultaneously.

Topographic wave drag tricky, since it acts differently on tidal versus non-tidal motions (Bell 1975)

Use bottom boundary-layer drag and topographic wave drag to derive energy dissipation ϵ, derive diffusivity κ from microstructure formula $\kappa = \frac{\Gamma\epsilon}{N^2}$

Examine feedback of mixing onto large-scale circulation.