Progress in embedding global tides into HYCOM simulations

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Am working closely with Alan on global forward tide modeling with HYCOM.

Builds on my experience using Hallberg Isopycnal Model (HIM) as a forward tide model.
Motivation

Tides provide significant fraction of energy available for mixing, which affects ocean general circulation.

General circulation affects generation and propagation of baroclinic (internal) tides, via stratification and Doppler-shifting.

Tides and general circulation couple nonlinearly through quadratic bottom drag and velocity advection terms.

Models of tides and non-tidal motions currently run separately—why not simultaneously?

Would like to have model including both, which derives mixing from dissipation, and dissipation from drag acting on both types of motions.

Target date of 2012 to deliver $1/25^\circ$ data-assimilative HYCOM model with tides. Want to begin with accurate forward tide model.
Starting with pioneering work (e.g., Hendershott 1972), we have learned over last 30+ years that optimally accurate forward tide models require:

• accurate astronomical forcing

• accurate solid earth body tides (direct response to astronomical forcing)

• accurate self-attraction and loading (SAL—gravitational self-attraction of ocean tide on itself, deformation and self-attraction of solid earth under ocean tidal load)

• tunable parameterizations of topographic wave drag acting in addition to nominal $c_d$ value of 0.0025

• accurate bathymetry and coastlines

• high horizontal resolution (at least 1/8°)
One-layer shallow-water equations

\[ \frac{\partial \eta}{\partial t} + \nabla \cdot [(H + \eta) \vec{u}] = 0 \]

\[ \frac{\partial \vec{u}}{\partial t} + (f + \zeta) \hat{k} \times \vec{u} = -g \nabla (\eta - \eta_{EQ} - \eta_{SAL}) \]

\[ -\nabla (\frac{1}{2} \vec{u} \cdot \vec{u}) + \nabla \cdot [K_H (H + \eta) \nabla \vec{u}] - \frac{c_d |\vec{u}| \vec{u}}{H + \eta} + \frac{\bar{T} \vec{u}}{\rho_0 (H + \eta)} \]

\( H \): resting water column thickness

\( \eta \): perturbation surface elevation

\( \vec{u} \): velocity

\( f \): Coriolis parameter

\( \zeta = \hat{k} \cdot (\nabla \times \vec{u}) \)

\( K_H \): horizontal friction

\( c_d \): quadratic drag coefficient

\( \bar{T} \): topographic drag tensor

\( \eta_{EQ}, \eta_{SAL} \): astronomical forcing, SAL
Astronomical forcing and body tides

• Semidiurnal tides ($M_2,S_2,N_2,K_2$):
  $$\eta_{EQ} = A(1 + k_2 - h_2)\cos^2(\phi)\cos(\omega t + 2\lambda),$$

• Diurnal tides ($K_1,O_1,P_1,Q_1$):
  $$\eta_{EQ} = A(1 + k_2 - h_2)\sin(2\phi)\cos(\omega t + \lambda),$$

where $\lambda$ is longitude wrt Greenwich, $\phi$ is latitude, $t$ is time wrt Greenwich, and $A$ and $\omega$ are constituent-dependent amplitudes and frequencies.

• $h_2$: accounts for solid-earth body tide deformation

• $k_2$: accounts for change in potential due to self-attraction of solid-earth deformation

• $(1 + k_2 - h_2) = 0.693$ for semidiurnal and long period tides, $= 0.736$ for diurnal tides due to “free-core nutation resonance” (Wahr 1981)

• Forcing frequencies are multiples, sums, and differences of natural frequencies in earth-moon-sun system—e.g., $S_2$ period is half a solar day (12 hours), $M_2$ period is half a lunar day (12.4 hours)
Self-atraction and loading

Earth yields to loading of ocean tide. Gravitational potential altered by self-attractions of mass redistributions in earth and ocean (Hendershott 1972):

$$\eta_{SAL} = \sum_n \frac{3\rho_{water}}{\rho_{earth}(2n+1)}(1 + k'_n - h'_n)\eta_n$$

$\eta_n$ n$th$ spherical harmonic of $\eta$

$k'_n$, $h'_n$ load numbers (Munk and MacDonald 1960) from Farrell (1972)

Solve by iteration, starting with “scalar approximation” $\eta_{SAL} \approx tidsal \ast \eta$.

Attaining convergence in iteration non-trivial, must use numerical tricks (Egbert et al. 2004; Arbic et al. 2004).
Topographic drag schemes–I

Models with only quadratic drag ($c_d=0.0025$) put all dissipation into shallow seas:

\[ < \rho_0 c_d |\mathbf{u}|^3 > = \]
\[ 0.02 \text{ mW m}^{-2}, |\mathbf{u}| = 2 \text{ cm s}^{-1}, \]
\[ 323 \text{ mW m}^{-2}, |\mathbf{u}| = 50 \text{ cm s}^{-1}. \]

Egbert and Ray (2000, 2001): T/P-constrained models yield $\sim 1$ TW dissipation over mid-ocean rough topography, in agreement with in-situ evidence (e.g. Polzin et al. 1997).

Jayne and St. Laurent (2001), Carrere and Lyard (2003), Egbert et al (2004), Arbic et al. (2004): accuracy of forward tidal models improved when topographic drag scheme is included alongside nominal $c_d$ value.
Topographic drag schemes–II

For HYCOM we use scheme of Arbic et al. 2004 and Garner (2005). Scheme builds on analytical result for drag on steady flow over arbitrary topography and includes scalings for nonlinear effects at bottom.

Have reduced tensor scheme $\frac{\bar{T}\bar{u}}{\rho_0(H+\eta)}$ to effective scalar field $r$ which operates on $\bar{u}$. Spatial average of $r$ very similar to spatial average of $r$ scheme used by Jayne and St. Laurent (2001).

Order 3-20 multiplicative factor "drgscl" must be applied and tuned to yield optimally accurate tides. Factor may account for lack of small scales in global topographic datasets, and for uncertain knowledge of internal wave break-
Dissipation in optimal $M_2$ runs with HIM

$\log_{10} M_2$ dissipation in W m$^{-2}$; Optimal $c_d$

$\log_{10} M_2$ dissipation in W m$^{-2}$; Optimal Topo Drag
Body tide forcing in HYCOM

Starting point was NCEP code for tidal body forcing.

Additions/alterations:

• \( \eta_{SAL} = \text{tidsal} \times \eta \)

• treat \( \eta_{SAL} \) exactly as astronomical forcing \( \eta_{EQ} \)

• allows for greater flexibility, i.e. for computing \( \eta_{SAL} \) iteratively later

• added topographic wave drag

• added option for either implicit, or CFL-limited explicit, bottom drag

• most HYCOM tide runs thus far have been with two layers (one having zero thickness)—32 layer runs just beginning

• most experiments \( M_2 \)-only: some with multi-constituents
Multi-constituent verification: HYCOM versus HIM

- First tested HYCOM on (almost) same 1/2° grid used for HIM

- RMS elevation errors (cm) from both models computed against GOT99 satellite altimetry model in waters deeper than 1000 m and equatorward of 66°.

- HYCOM performs quite well, although proper iterations of $\eta_{SAL}$ and proper harmonic analysis of tidal frequencies not yet in place.

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Signal</th>
<th>HIM (HYCOM) errors</th>
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<tbody>
<tr>
<td>Q₁</td>
<td>1.39</td>
<td>0.36 (0.59)</td>
</tr>
<tr>
<td>O₁</td>
<td>6.61</td>
<td>1.57 (2.36)</td>
</tr>
<tr>
<td>P₁</td>
<td>3.13</td>
<td>0.77 (0.80)</td>
</tr>
<tr>
<td>K₁</td>
<td>9.54</td>
<td>2.45 (2.51)</td>
</tr>
<tr>
<td>N₂</td>
<td>5.65</td>
<td>1.51 (2.01)</td>
</tr>
<tr>
<td>M₂</td>
<td>26.69</td>
<td>7.76 (8.00)</td>
</tr>
<tr>
<td>S₂</td>
<td>10.57</td>
<td>4.26 (3.56)</td>
</tr>
<tr>
<td>K₂</td>
<td>2.97</td>
<td>1.08 (1.09)</td>
</tr>
</tbody>
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HYCOM tide model sensitivities

Have tested HYCOM tide model sensitivity to:

• value of scalar used in scalar approximation to self-attraction and loading

• strength of topographic wave drag

• bathymetric grid used
  – HYCOM grid a0.72 (derived from DBDB5)
  – new HYCOM grid t0.72 (derived from DBDB2)
  – HYCOM grid a0.08 (derived from DBDB2)

Some testing of numerics such as length of time step, and usage of implicit versus explicit schemes for drag. Numerics of time-splitting will also surely matter.
Sensitivity to self-attraction and loading

On $1/2^\circ$ grid, HYCOM yields lower RMS $M_2$ errors when $\eta_{SAL}=0.06\eta$. HIM better with $0.094\eta$. Latter value is from least squares fit in waters deeper than 1000 m of correctly computed $\eta_{SAL}$, versus $\eta$, in GOT99 altimetry.
Maps of RMS $M_2$ errors (cm)—HYCOM versus GOT99 altimetry

- Old $0.72^\circ$ grid; $c_d$ only: avg error 52.54 cm
- Old $0.72^\circ$ grid; drgsc=12.6; avg error 15.93 cm
- New $0.72^\circ$ grid; $c_d$ only: avg error 25.12 cm
- New $u./z^\circ$ grid; drgsc=12.6; avg error 9.95 cm
- $0.08^\circ$ grid; $c_d$ only; avg error 17.54 cm
- $0.08^\circ$ grid; drgsc=12.0; avg error 6.70 cm
RMS $M_2$ errors—t0.72 versus a0.08

New 0.72° grid; drgscI=12.6; avg error 9.93 cm

0.08° grid; drgscI=12.0; avg error 6.70 cm
Multi-constituent t0.72 HYCOM vs pelagic tide gauges
Where next?

- Target: $1/12^\circ$ multi-layer run with wind, buoyancy, and tidal forcing

- Issue: how to handle SAL in wind- plus tides runs? (may be very different for two classes of motions)

- Issue: how to handle topographic wave drag in wind- plus tides runs (drag physics differ for two motions)

- Examine effects of tides on general circulation (i.e. Indonesian throughflow), and vice versa

- Use bottom boundary-layer drag and topographic wave drag to derive energy dissipation $\epsilon$, derive diffusivity $\kappa$ from $\epsilon$, examine feedback of mixing onto large-scale circulation.

- Utilize better bathymetry maps (ONR project with John Goff, U-Texas)

- Examine impact of bottom drag on eddies, role of eddies in mixing (with Bill Schmitz, NRL collaborators, Rob Scott of U-Texas)