Implicit Solution of the Vertical Diffusion Equation for the Reynolds Stress Mixing Models

The solution procedure in HYCOM follows the procedure used by Large et al. (1994) and is used for all three Reynolds stress vertical mixing models (KPP, GISS, MY). Decomposing model variables into mean (denoted by an overbar) and turbulent (denoted by a prime) components, the vertical diffusion equations to be solved for potential temperature, salinity, and vector momentum are

\[ \frac{\partial \bar{\Theta}}{\partial t} = -\frac{\partial}{\partial z} w' \Theta', \quad \frac{\partial \bar{S}}{\partial t} = -\frac{\partial}{\partial z} w' S', \quad \frac{\partial \bar{v}}{\partial t} = -\frac{\partial}{\partial z} w' v'. \]  \hspace{1cm} (1)

Boundary layer diffusivities and viscosity parameterized as follows:

\[ w' \Theta' = -K_\theta \left( \frac{\partial \bar{\Theta}}{\partial z} + \gamma_\Theta \right), \quad w' S' = -K_s \left( \frac{\partial \bar{S}}{\partial z} + \gamma_s \right), \quad w' v' = -K_m \left( \frac{\partial \bar{v}}{\partial z} + \gamma_m \right), \]  \hspace{1cm} (2)

where the \( \gamma \) terms represent nonlocal fluxes. For example, the KPP model includes nonlocal terms for \( \Theta \) and \( S \), but not for momentum. The following solution procedure is valid for any mixing model in HYCOM that calculates the diffusivity/viscosity profiles at model interfaces, whether or not nonlocal terms are parameterized.

The following matrix problems are formulated and solved:

\[ A_t \Theta^{t+1} = \Theta' + H_\Theta \quad A_s S^{t+1} = S' + H_s \quad A_M M^{t+1} = M' + H_M, \]  \hspace{1cm} (3)

where superscripts \( t, t+1 \) denote model times, and \( M \) is the vector of a momentum component, either \( u \) or \( v \). The matrices \( A \) are tri-diagonal coefficient matrices, while the vectors \( H_t \) and \( H_s \) represent the nonlocal flux terms. Given \( K \) model layers with nonzero thickness, where an individual layer \( k \) of thickness \( \delta p_k \) is bounded above and below by interfaces located at pressures \( p_k \) and \( p_{k+1} \), the matrix \( A_s \) is determined as follows:

\[ A_s^{1,1} = \left( 1 + \Omega_{s1}^+ \right) \quad A_s^{k,k-1} = -\Omega_{sk}^- \quad 2 \leq k \leq K \]
\[ A_s^{k,k} = \left( 1 + \Omega_{sk}^- + \Omega_{sk}^+ \right) \quad 2 \leq k \leq K \]
\[ A_s^{k,k+1} = -\Omega_{sk}^+ \quad 1 \leq k \leq K - 1 \]  \hspace{1cm} (4)

with

\[ \Omega_{sk}^- = \frac{\Delta t}{\delta p_k} \frac{K_S(p_k)}{p_{k+0.5} - p_{k-0.5}} \]
\[ \Omega_{sk}^+ = \frac{\Delta t}{\delta p_k} \frac{K_S(p_{k+1})}{p_{k+1.5} - p_{k+0.5}} \]  \hspace{1cm} (5)

where \( p_{k+0.5} \) represents the central pressure depth of model layer \( k \). The nonlocal flux arrays are calculated using
\[
H_{s1} = \frac{\Delta t}{\delta p_1} K_S(p_{k+1}) \gamma_S(p_{k+1})
\]

\[
H_{s\kappa} = \frac{\Delta t}{\delta p_k} \left[ K_S(p_{k+1}) \gamma_S(p_{k+1}) - K_S(p_k) \gamma_S(p_k) \right] \quad 2 \leq k \leq \kappa.
\]

The solution is then found by inverting the tri-diagonal matrix \( A \). The matrix problems are formulated and solved in the same manner for potential temperature and momentum components.

**REFERENCE**